

Transient Carrier Frequency Offset by using Adaptive Weighted Subspace Fitting Algorithm in Wireless Transceivers

Mr. Shahane S.B.¹, Mr. Admane R.K.², Mrs. Telure P.R.³

¹Head of the Department of Electronics and Telecommunication Mitthulalji Sarda Polytechnic College Beed, India. ²Lecturer of the Department of Electronics and Telecommunication Mitthulalji Sarda Polytechnic College Beed, India ³Lecturer of the Department of Electronics and Telecommunication Mitthulalji Sarda Polytechnic College Beed, India

Abstract - Future remote gadgets need to backing numerous applications (e.g., the remote apply autonomy, remote computerization, and versatile gaming) with amazingly low delay and reliability over wireless connection. Previously, many hardware impairments that have been optimizing wireless transceivers while switching between wireless connections with different circuit characteristics requires. То encourage TDD, exchanging between transmissions and gathering radio capacities can change load on the force supply. As the supply voltage changes in light of the sudden change in load, drifts the carrier frequency. Such the result of drift transient carrier frequency offset (CFO) that can't be assessed by ordinary CFO estimators and the guard intervals are typically addressed by inserting or extending. This paper proposes Adaptive Weighted Subspace Fitting (AWSF) algorithm to achieve minimized Mean Square Error (MSE) performance than in the simple Weighted Subspace Fitting (WSF) method.

Key Words: carrier frequency offset (CFO) Adaptive Weighted Subspace Fitting (AWSF).

1.INTRODUCTION

Many improvements from air interface design to signal processing techniques that mitigate radio frequency impairments of optimizing wireless physical layer for capacity and reliability. for example, the radio frequency (RF) impairment in wireless systems is well understood the carrier frequency offset (CFO) [2].the time division duplex (TDD) is deployed as the duplexing scheme because the impact of the transient CFO can be more pronounced in wireless systems.

It must be taken into account in applications such as remote robotics, wireless automation, and mobile gaming where extremely low latency and reliability requirements have to be met over connections with future use of 5G or legacy wireless systems. Such systems include the next generation mobile communication systems, Long-Term Evolution (LTE) with TDD mode [2][4], and cooperative communications where TDD mode is operated in cooperative relay transmissions [5][6].the designed TDD LTE with guard period between the switching from the downlink to the uplink. Instead of dealing with the transient effect the guard period is given to guarantee that user equipment (UE) can switch between reception and transmission with no overlap of signals. To handle the propagation delay in the cells the length of the guard period is designed. The design of Hardware is more challenging for UE away from base station [2] [3] [7]. Wireless sensor networks (WSN) is another example, which is require robust wireless communication protocol with low dormancy and power consumption, switching between transmission and reception and low duty cycle are broadly designed in MAC layer protocol of WSN[8]-[10]. In this paper we are considering the problem formulation and parametric estimation as proposed in the previous work [1]. We provide the performance analysis of given system using transient carrier frequency offset and present the performance analysis to minimized MSE and results are obtained using extensive numerical calculations on MATLAB.



The rest of the paper is organized as follows: in section II, the motivation of correcting the transient CFO is presented as well as its signal model. The model of the additive noise in the phase difference is also discussed. In section III, The proposed estimation algorithms based on subspace are presented the weighted subspace fitting algorithm is presented after we give the perturbation analysis. Numerical examples and experimental results are reported in Section IV. Conclusions are drawn in Section V.

2. PROBLEM FORMULATION

A. The Traditional CFO Estimation

The transmitted preamble signal can be written as, by assuming the receiver estimates the CFO through the preamble OFDM symbol, whose first and second halves are identical[1].s(t) =

$$\begin{cases} s_1(t) & 0 \le t \le T \\ S_1(t-T) & T \le t \le 2T \end{cases}$$
(1)

Where T is the interval between the two halves. Thus, the received signal impaired by CFO can be written as,

$$r(t) = \left(\int_{-\infty}^{+\infty} h(v)s(t-v)dv + n(t)\right)e^{\int_{0}^{t} j\Delta\omega(\tau)d\tau} + n(t)$$
(2)

Where the channel response is h (t); the transmitted preamble is s (t); and the additive Gaussian noise at the receiver before and RF down conversion is n(t) and $\hat{n}(t)$, respectively. $\Delta \omega(t)$ is the CFO between the transmitter and receiver. $\hat{n}(t)$ Is much less than n (t). Hence, thereafter noise term is drop. We assume that the CFO is constant, for the traditional CFO estimation, and then (2) reduces to

$$r(t) = \left(\int_{-\infty}^{+\infty} h(v)s(t-v)dv + n(t)\right)e^{j\omega t} 0 \le t \le 2T \quad (3)$$

The received preamble can be written as the first and second halves as,

$$r_{1}(t) = \left(\int_{-\infty}^{+\infty} h(v)s(t-v)dv + n_{1}(t)\right)e^{j\Delta\omega t}$$
$$0 \le t \le T$$
$$r_{2}(t) = \left(\int_{-\infty}^{+\infty} h(v)s(t-v)dv + n_{2}(t)\right)e^{j\Delta\omega(t+T)}$$
$$0 \le t \le T$$

Where, the additive noise is $n_1(t)$ and $n_2(t)$ in $r_1(t)$ and $r_2(t)$, respectively. Therefore, the received preamble symbols have a fixed phase difference of $\Delta\omega T$ in sample-wise with the absence of the additive noise of first and second halves. To compute this fixed phase Difference of the traditional CFO estimation. The maximum likelihood estimation of the CFO, denoted by $\Delta\hat{\omega}$, is (1)

$$\Delta \widehat{\omega} = \frac{\langle (\sum_{n=0}^{N-1} r_1^*(nT_s)r_2(nT_s))}{T}$$
(4)

Where the received signal samples of the first and second halves of the preamble symbol $r_1(nT_s)$ and $r_2(nT_s)$.sampling interval is T_s , preamble symbol of each half of length is N.

B. Experimental Characterization of CFO

We have set up using two terminals of traditional CFO estimation algorithm presented above has been implemented on the test bed. National Instruments (NI) FlexRIO family modules and the Ettus USRP RF front-ends of combine each terminal as shown in fig.1 (a).

We use the modulation technique of orthogonal frequency division multiplexing (OFDM) for our physical layer design. In fig.2 we are using the time division duplexing (TDD) is operated with a frame structure. The beginning of each frame, the master terminal transmits the preamble OFDM symbol and four regular OFDM symbols are followed. Similarly, after TX to RX switching, the preamble and four regular OFDM symbols continuously receives the slave terminal and it updates the slave terminal of time and frequency synchronization through preamble symbol [1].

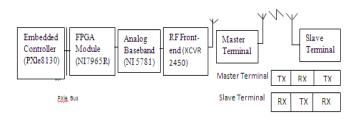


Fig1. (a) Hardware architecture of a test bed terminal. (b) System diagram of the test bed in real-time TDD operation.



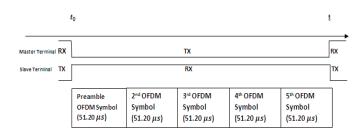


Fig 2: Frame structure t_0 is the time when the master terminal switches from RX to TX and when the slave terminal switches from TX to RX.

The traditional CFO estimator has a bias of 1000 Hz over the preamble OFDM symbol opposed to the anticipated 0Hz CFO. The estimated CFO by the traditional CFO estimator decays with time. [1]. in the second OFDM symbol the output of traditional CFO estimator reduces to 65 Hz and the fourth and fifth OFDM symbols even reduces to less than 10Hz. The second to the fifth OFDM symbols of estimated CFOs are accessed by computing the phase shift between the end parts of the OFDM symbol and received cyclic prefix.As observed and extensively documented in, this phenomenon can occur in compact and highly integrated RF transceivers that share a common power supply across the transmission and receive functions [1].

C. Transient CFO Model

To develop algorithms to estimate and compensate the transient CFO introduced in compact RF transceivers in our experiments and similar findings in Rice WARP boards have motivated us [2]. The voltage response of dc-dc converter can be approximated when step change in load current of second order control system [12].we requirement that he transient CFO, denoted by $\Delta \omega_T(t)$, can be model as the step response of a second order under damped system, can be expressed as an exponentially damped sinusoid [1], i.e.,

$$\Delta \omega_T(t) = \alpha e^{-\zeta \omega_n t} \operatorname{Sin}(\omega_n \sqrt{1 - \zeta^2 t + \varphi}) \ t > 0, \ 0 < \zeta < 1 \ (5)$$

Where ζ is damping factor and ω_n under damped natural frequency, the second order system is under damped that means $0 < \zeta < 1$ [1]. Initial phase is φ and gain of response is α respectively. The transient CFO model, defined two parts of overall CFO: steady state CFO and transient CFO. We assume that the steady state CFO, denoted by $\Delta \omega_s$, is constant during the preamble OFDM symbol. Then the overall CFO can be

written as $\Delta \omega(t) = \Delta \omega_T(t) + \Delta \omega_S$.thus, the received signal becomes

$$r(t) = \left(\int_{-\infty}^{+\infty} h(v)s(t-v)dv + n(t)\right)e^{\int_{0}^{t} j(\Delta\omega_{T}(\tau) + \Delta\omega_{S})d\tau}$$
(6)

In this paper, consider that the steady state CFO is estimated and removed from the baseband signal to simplify the estimation of the transient CFO [1].

Let us now suppose the two halves in received preamble symbol

$$r_1(t) = \left(\int_{-\infty}^{+\infty} h(v)s(t-v)dv + n_1(t)\right)e^{\int_0^t j\Delta\omega_T(\tau)d\tau}$$
$$r_2(t) = \left(\int_{-\infty}^{+\infty} h(v)s(t-v)dv + n_2(t)\right)e^{\int_0^{t+T} j\Delta\omega_T(\tau)d\tau}$$

The phase difference between the first and second halves is denoted by $\Psi(t)$

$$\Psi(t) = < \left(r_1^*(t)r_2(t)\right) = \int_t^{t+T} \Delta\omega_T(\tau)d\tau + n_{\Psi}(t)$$
(7)

Where, the noise term in the phase difference is $n_{\psi}(t)$.

Note that $\Psi(t)$ is just the definite integral of the transient CFO over interval [t, t + T]. Then $\Psi(t)$ is still an exponentially damped sinusoid with the same damping factor and frequency, but different initial phase and gain, i.e.

$$\Psi(t) = \int_{t}^{t+T} \Delta \omega(\tau) d\tau + n_{\Psi}(t)$$

$$=\alpha_{\Psi}e^{-\zeta\omega_{n}t}\sin(\omega_{n}\sqrt{1-\zeta^{2}t}+\phi_{\Psi})+n_{\Psi}(t)$$
(8)

3. PARAMETRIC ESTIMATION

A. Subspace Based Estimation of a and ω_s

The existing algorithms are, however, rather generic and can be generally put into four categories: direct fitting in time domain using signal samples, direct fitting in frequency domain with DFT based algorithms, covariance based linear prediction, and subspace (SVD) based linear prediction . Then, the complex roots of the prediction polynomial are found to compute the damping factor and frequency. International Journal of Scientific Research in Engineering and Management (IJSREM)Volume: 07 Issue: 04 | April - 2023Impact Factor: 8.176ISSN: 2582-3930

$$\Psi_{r} = \begin{bmatrix} \Psi(0) & \Psi(r) & \Psi(2r) \\ \Psi(1) & \Psi(r+1) & \Psi(2r+1) \\ \vdots & \vdots & \vdots \\ \Psi(N-2r-1) & \Psi(N-r-1) & \Psi(N-1) \end{bmatrix}$$
(9)

As a Hankel-like matrix, where r is a positive integer that denotes the time lag to be optimized later [1].

B. Performance Analysis and Weighted Subspace Fitting

We purpose the first order subspace perturbation analysis. Where Hankel-like matrix is $\Psi_r = H_r + W_r$, where Ψ_r is defined in (9), H_r is the noiseless signal matrix and W_r is a Hankel-like matrix composed of noise, i.e.,

$$\Psi_r = \begin{bmatrix} n_{\Psi}(0) & n_{\Psi}(r) & n_{\Psi}(2r) \\ n_{\Psi}(1) & n_{\Psi}(r+1) & n_{\Psi}(2r+1) \\ \vdots & \vdots & \vdots \\ n_{\Psi}(N-2r-1) & n_{\Psi}(N-r-1) & n_{\Psi}(N-1) \end{bmatrix}$$
(10)

The singular value decomposition (SVD) of H_r is

$$H_r = \begin{bmatrix} U_s & U_0 \end{bmatrix} \begin{bmatrix} \Sigma_s & 0\\ 0 & 0 \end{bmatrix} \begin{bmatrix} V^H\\ v_0^{H^s} \end{bmatrix} = U_s \Sigma_s V_s^H$$
(11)

Then, the null space of H_r is spanned by $v_0 = [v_{00}, v_{01}, v_{02}]^T$. Thus, $e^{\alpha r \pm j\omega_s r}$ is two zeros of prediction polynomial

$$B(z) = v_{00} + v_{01}z^{-1} + v_{02}z^{-2}$$
(12)

To improve the estimation accuracy by the truncated SVD approximation using the existing subspace based methods, like the KT method and matrix pencil method [20]. The SVD of H_r is defined in (11). Consider that the SVD of Ψ_r is given by,

$$\Psi_r = \begin{bmatrix} \widetilde{U_s} & \widetilde{U_0} \end{bmatrix} \begin{bmatrix} \widetilde{\Sigma_s} & 0\\ 0 & \widetilde{\Sigma_0} \end{bmatrix} \begin{bmatrix} \widetilde{V_s}^H\\ \widetilde{v}_0^H \end{bmatrix} = \widetilde{U}_s \widetilde{\Sigma}_s \widetilde{V}_s^H + \widetilde{U}_0 \widetilde{\Sigma}_0 \widetilde{v}_0^H$$
(13)

At that point, analyzing (11) and (13), we determine the estimation error $\Delta v_0 = v_0 - \tilde{v}_0$ as the perturbation of v_0 .as per, estimation of the first-order for perturbation at high SNR is given by

$$\Delta v_0 \approx -V_s \Sigma_s^{-1} U_s^H W_r v_0 \tag{14}$$

It is clear that $E[\Delta v_0] = 0$ since $E[W_r] = 0$. we also have the mean square error (MSE) of v_0

$$\mathbb{E}[\|\Delta v_0\|^2] = Tr(U_S \Sigma_s^{-2} U_s^H \Gamma)$$
(15)

Where Γ is characterized as $E[\Delta v_0]$. Γ is a scanty grid with non-zero sections on five diagonals signified by

$$[\Gamma]_{i,i} = \frac{|v_{00}|^2}{\rho((i-1)T_s)} + \frac{|v_{01}|^2}{\rho((i+r-1)T_s)} + \frac{|v_{02}|^2}{\rho((i+2r-1)T_s)}$$
(16)

$$[\Gamma]_{i,i+r} = \frac{v_{00}^* v_{01}}{\rho((i+r-1)T_S)} + \frac{v_{01}^* v_{02}}{\rho((i+2r-1)T_S)}$$
(17)

$$[\Gamma]_{i,i+2r} = \frac{v_{00}^* v_{02}}{\rho((i+2r-1)T_s)}$$
(18)

Now we have $[\Gamma]_{i+nr,i} = [\Gamma]_{i,i+nr}^*$, n=1, 2 due to the Hermitian covariance matrix Γ we propose weighted subspace fitting technique so as to reduce the MSE of the estimated v_0 . The weighted subspace fitting can be formed as

$$\Psi_{rD} = D\Psi_r = DH_r + DW_r \tag{19}$$

$$\Psi = \begin{bmatrix} \Psi(0) & \Psi(1) & \cdots & \Psi(L-1) \\ \Psi(1) & \Psi(2) & \cdots & \Psi(L) \\ \vdots & \vdots & \ddots & \vdots \\ \Psi(N-L-2) & \Psi(N-L-1) & \cdots & \Psi(N-1) \end{bmatrix}$$
(20)

L order prediction filter, which is also the number of columns of the Hankel matrix, is chosen to optimize the estimation performance in the subspace based linear prediction problems, such as the kumaresan-Tuft (KT) method.

C. Selection of Time Lag r

To construct a noiseless signal matrix H_r , firstly the time lag r must be evaluated. The following are the step to estimate time lag r. The mean square error of Δv_D is defined as,

(a)

$$\mathbf{E}[\|\Delta v_D\|^2] = T_r((U_s^H \Gamma^{-1} U_s)^{-1} \Sigma_s^{-2})$$

$$\leq T_{r}((U_{S}^{H}\Gamma^{-1}U_{S})^{-1})T_{r}(\Sigma_{S}^{-2})$$
(b)

$$\leq \sum_{n=0}^{N-1} \frac{1}{\rho(nT_{S})} \frac{\sigma_{1}^{2} + \sigma_{2}^{2}}{\sigma_{1}^{2}\sigma_{2}^{2}} \qquad (21)$$

The mean square error is defined in three steps. In step (a) both $(U_s^H \Gamma^{-1} U_s)^{-1}$ and \sum_s^{-2} are positive semi definite. In step (b) both σ_1 and σ_2 are two non-zero singular values of H_r in term of r. the sample covariance matrix of noise free matrix H_r is asymptotically equivalent to,

$$\lim_{N \to \infty} \frac{1}{N - 2r} H_r^H H_r = F_3^{(r)} F_3^{(r)H}$$
(22)



Where
$$F_3^{(r)}$$
 is a Vander monde matrix defined as,

$$F_{k}^{1} = \begin{bmatrix} 1 & 1 \\ e^{(a+jw_{s})l} & e^{(a-jw_{s})l} \\ \vdots & \vdots \\ e^{(k-1)(a+jw_{s})}l & e^{(k-1)(a-jw_{s})}l \end{bmatrix}$$
(23)

D. Estimation of the Initial Phase and Gain

We can easily estimate initial phase as well as the system gain with the help of least squares method, if we know the values of the \hat{a} and $\hat{\omega}_s$. The least square problem is given as below,

Observation vector $\Psi = [\Psi(0), \Psi(1), ..., \Psi(N-1)]^T$ the least square estimation of r is $\hat{r}, \hat{r} = (F_N^{(1)H} F_N^{(1)})^{-1} F_N^{(1)H} \Psi$

Also we know $\mathbf{r}, \mathbf{r} = \left[\frac{\alpha_{\Psi}e^{j\varphi_{\Psi}}}{2j}, -\frac{\alpha_{\Psi}e^{-j\varphi_{\Psi}}}{2j}\right]^{T}$

Positive definite matrix can be calculated as shown below when we know the covariance matrix of φ is

$$\mathbf{R}, \mathbf{R}^{-1} = S^H S \tag{24}$$

With the estimation of \hat{r} , we can further calculate the estimation of initial phase, $\hat{\varphi}$, and system gain, \hat{a} , in

$$\hat{r} = ((SF_N^{(1)})^H SF_N^{(1)})^{-1} (SF_N^{(1)})^H S\Psi$$
(25)

E. Transient CFO by using Adaptive weighted subspace fitting algorithm

An adaptive filter is a filter containing coefficients that are updated by some type of adaptive algorithm to improve or somehow optimize the filter's response to a desired performance criterion. In general, adaptive filters consist of two basic parts: the filter which applies the required processing on the incoming signal which is to be filtered; and an adaptive algorithm, which adjusts the coefficients of that filter to somehow improve its performance.

The filter output signal y (n) is given by,

$$Y(n) = W^{T}(n)X(n) = \sum_{l=0}^{N-l} W_{i}(n)X(n-l)$$
(26)

Where, $\underline{X}(n) = [x(n)x(n-1)...x(n-N+1)]^T$ is the input vector, $\underline{W}(n) = [W_0(n)W_1(n)...W_{N-1}(n)]^T$ Is the weight vector, T denotes Transpose, n is the time index, N is the order of filter. This example is in the form of a finite impulse response filter as well as the convolution (inner product) of the two vectors $\underline{x}(n)$ and $\underline{w}(n)$. The adaptation algorithm uses the error signal,

$$e(n)=d(n)-y(n)$$
 (27)

Where d (n) is the desired signal and y (n) is the filter output. The input vector x (n) and e (n) are used to update the adaptive coefficients according to a criterion that is to be minimized. The criterion employed in this section is the mean-square error (MSE) \in :

$$\equiv = \mathbb{E}\left[e^2(n)\right] \tag{28}$$

Where E [.] denotes the expectation operator. If y (n) from Equation (25) is substituted into Equation (26), then Equation (27) can be expressed as,

$$\in = \mathbb{E}[d^2(n)] + \underline{W}^T(n)R\underline{W}(n) - 2\underline{W}^T(n)P \qquad (29)$$

4. EXPERIMENTAL RESULTS

In this work, we propose results via numerical calculations on MATLAB platform. In the above figure of time vs. signal power is represented. Black line represents the estimated signal and the blue color denotes the variations in the original signal.

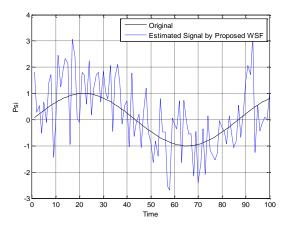


Fig 6: Comparison between original and estimated signals.



Impact Factor: 8.176

ISSN: 2582-3930

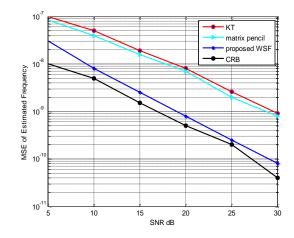


Fig7. Comparison of the MSE of the estimated frequency

In this figure 7.represents the signal to noise ratio vs. MSE of the damping factor for the proposed method and compared with other existing techniques. The methods compared include the proposed subspace methods with WSF, the KT method and the matrix pencil method. The CRB for the estimated damping factor and frequency with colored noise and above fig also shows comparison of Adaptive filter which recursively finds the coefficients that minimizes a weighted linear least squares cost function relating to the input signals. This is the contrast to other algorithms such as the least mean square (LMS) that aim to reduce the mean square error.

Fig 8 Shows the comparison of the detection SNR of the received preamble OFDM symbol with different processing schemes: without correcting the transient CFO, with correcting the transient CFO by the estimation algorithm with and without the proposed WSF. We can see that the detection SNR with removing the transient CFO is much higher than that without removing the transient CFO as shown in fig. (a) And we can see that the improvement of the detection SNR is significant when the transient CFO is removed from the preamble OFDM symbol as shown in fig. (b).

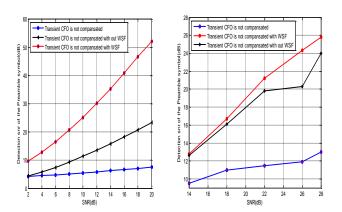


Fig8. (a) And (b) Comparison of the detection SNRs of the received symbols with different processing schemes.

5. CONCLUSIONS

The proposed work called evaluation of transient CFO by using adaptive weighted subspace fitting algorithm in wireless Transceivers. We have analyzed a unique problem in the wireless communication system i.e. CFO (carrier frequency offset). It was mainly observed in switching systems of transmitter and receiver. To reduce this problem we proposed an algorithm based on the subspace decomposition of the Hankel-like matrix and to improve the Estimation accuracy adaptive weighted subspace fitting algorithm is proposed. It has been shown via numerical calculations on MATLAB platform that proposed work provides minimize MSE (Mean Square Error) over classical WSF (Weighted subspace fitting).

The transient impairments would cause more concern in future wireless devices, which have to support many applications (e.g., remote robotics, wireless automation, and mobile gaming) with extremely low latency and reliability requirements over wireless connections.

Acknowledgment

We would like to thank anonymous referees for their valuable comments and suggestions to improve the quality of paper.



References

- Jin Yuan," Modeling and Estimation of Transient Carrier Frequency Offset in Wireless Transceivers", IEEE TRANSACTIONS ON WIRELESS COMMUNICATIONS, VOL. 13, NO. 7, JULY 2014.
- P. O. Murphy, "Design, implementation and characterization of a cooperative communications system," Ph.D thesis, Rice University, Houston, TX, USA, 2010.
- Evolved Universal Terrestrial Radio Access (E-UTRA); Physical Channels and Modulation, 3GPP TS 36.211 version 9.1.0 Release 9,Mar. 2010.
- Evolved Universal Terrestrial Radio Access (E-UTRA); Physical layer procedures, 3GPP TS 36.213 version 10.1.0 Release 10, 2011.
- A. Ghosh, R. Ratasuk, B. Mondal, N. Mangalvedhe, and T. Thomas, "LTE-Advanced: Next-generation wireless broadband technology," IEEE Wireless Commun., vol. 17, no. 3, pp. 10–22, Jun. 2010.
- 6. G. Bradford and J. Laneman, "A survey of implementation efforts and experimental design for cooperative communications," in Proc. IEEEICASSP, Dallas, TX, USA, Mar. 2010, pp. 5602– 5605
- 7 . G. Kramer ,I. Mari'c,andR.Yates,Cooperative Communications, Foundations and Trends in Networking, vol. 1. Princeton, NJ, USA: PrincetonUniv. Press, 2006, No. 3
- D. Astélyet al., "LTE: The evolution of mobile broadband," IEEE Commun. Mag., vol. 47, no. 4, pp. 44–51, Apr. 2009
- G. Lu, B. Krishnamachari, and C. S. Raghavendra, "An adaptive energy efficient and low-latency MAC for data gathering in wireless sensor networks," in Proc. 18th Int. Parallel Distrib. Process. Symp., Apr. 2004, p. 224.
- W. Ye, J. Heidemann, and D. Estrin, "Medium access control with coordinated adaptive sleeping for wireless sensor networks," IEEE/ACM Trans.Netw., vol. 12, no. 3, pp. 493–506, Jun. 2004.
- 11.P. H. Moose, "A technique for orthogonal frequency division multiplexing frequency offset correction,"*IEEE Trans. Communications, vol. 42,no. 10, pp. 2908–2914, Oct. 1994.*

 P. Zielinski and K. Duda, "Frequency and damping estimation methods—An overview," *Metrology Meas. Syst.*, vol. 18, no. 4, pp. 505–528, Dec. 2011.