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TWO-DIMENSIONAL ARRAY CASUSED BY ABLOWITZ-LADIK-HIROTA WAVE GUIDE COMPLEXITY

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ABSTRACT

We prove that the rational solutions exist for the integrable version of the discretized nonlinear Schrödinger equation known as the Ablowitz-Ladik equation, which exhibits rogue wave behavior. We establish the existence of a pecking order among rational solutions, and then use the Hirota method to deduce the first two solutions in that order. The stability of discrete similaritons and first-order rogue waves propagating across nonlinear waveguide arrays may be simulated by means of the inhomogeneous ALH equation. The similarity transformation is used to find these nonlinear solutions. Complexity and stability in discrete or continuous spatial configurations are the focus of CC, a CIM. When the amplitude modulation parameter is set to illustrative values, we discovered that the CC demonstrates saturation for a longer duration for rogue waves and a shorter time for free waves. We also provide rational solutions for the more general discrete Hirota equation, which includes the discrete Ablowitz-Ladik equation and the discrete modified Korteweg-de Vries mKdV equation as special instances.

KEYWORDS: Ablowitz-Ladik equations; Hirota bilinear method; Discrete breathers; Rogue waves.

INTRODUCTION

Important physical phenomena, rogue waves are spatiotemporally limited and represent an isolated occurrence with no preceding or following examples. If a rogue wave (RW) reappears immediately after an encounter with no discernible change in size or form, we refer to it as a Rogan. The ocean, nonlinear optics, Bose-Einstein condensates the atmosphere, and even the financial sector have all been shown to include RWs. The existence and usefulness of optical RWs in nonlinear optical fibers has also been shown by a few experimental studies. Optical RWs are distinct from the potentially disastrous oceanic RWs, which have been reported to affect passenger ships, container ships, oil tankers, fishing boats, and offshore and coastal buildings with often devastating results. In particular, the analytical RWs have been obtained for some physical models, including the shallow water Kadomtsev-Petviashvili equation and the nonlinear Schrödinger (NLS) equation, as well as some of their extensions with varying coefficients, higher orders, or higher dimensions.

Several areas of physics, such as fluid mechanics and optics, may benefit from understanding the history and dynamics of wave packets as regulated by hierarchies of nonlinear Schrödinger equations. In addition to their inherent importance in theoretical physics, discrete versions of these nonlinear equations have been investigated extensively because they describe genuine practical scenarios, such as spatially confined modes in a periodic array of optical waveguides. The Ablowitz-Ladik system for oscillators on an integer lattice (t = time, n = integer, * = complex conjugate) is an example of an evolution equation that permits analytical progress.

$$iu_{n,t} + \beta(u_{n+1} + u_{n-1} - 2u_n) + \sigma u_n u_n^*(u_{n+1} + u_{n-1}) = 0, u_n = u_n(t).$$

A brief word on Ablowitz-Ladik systems' physics-related uses is in order. One difficulty is the conflict between nonlinearity and randomness; for example, a soliton's stability feature may be destroyed or severely degraded by a random potential. The dynamics of discrete solitons as they travel over a Möbius strip of two connected arrays is another example of this kind of use. Potential topological switches and monopole spectra in parameter space are well-represented by the Ablowitz-Ladik system. The switching time at which soliton modes transition from one array to the next may be precisely controlled by a linear interchain coupling.

The purpose of this paper is to theoretically suggest a new kind of linked Ablowitz-Ladik system with both SPM and XPM. In this case, we develop the Hirota bilinear form. Under some circumstances, we find exact periodic (breather) and localized (rogue wave) solutions. The presence of conservation rules is tested using a single spatially periodic solution.

From physics and chemistry to the biological sciences and engineering, nonlinear phenomena may be found in every branch of study. Nonlinear evolution equations are a kind of partial differential equation used to represent nonlinear systems in many practical contexts (NLEEs). Solutions in the form of spatially confined excitations are of great interest. Several of these excitations are weakly radiated or nonradiative configurations that preserve their original form across very long distances. This unique quality is what gives rise to the name "solitons" or "solitary waves" to describe these arrangements. Several scientific disciplines, from hydrodynamics and plasma physics to condensed matter physics and optical communications to nuclear physics and astrophysics, have found solitons and solitons-like solutions.

LITREATURE REVIEW

Jun Yang (2022) In this study, we look at how the generalized Darboux transformation affects the solutions of a generalized integrable discrete nonlinear Schr odinger (NLS) problem, namely the smooth positon and breather-positon solutions (DT). Degenerate DT is used to generate Nth-order smooth positon solutions from the initial zero-seed solution. From this nonzero seed solution come the many breather solutions, such as the Akhmediev breather, the Kuznetsov-Ma breather, and the spacetime periodic breather. The eigenfunctions of breather solutions are then gradually expanded using a Taylor series, and this yields the breather-positon solutions. We analyze the impact of higher-order nonlinear terms on these discrete smooth positon solutions and breatherpositon solutions, showing that the interacting region of soliton-positon and breatherpositon is highly compressed by higher-order nonlinear effects, but the distance between the two positons has an opposite effect in two waveforms.

Yu, Fajun. (2015). Using the extended Darboux transformation, we investigate multi-rogue wave solutions to a Schrodinger equation with higher-order components. Using the combined Hirota-Lakshmanan-Porsezian-Daniel (LPD) equation, we conduct an analytical study of several features of the nonautonomous rogue waves. We think about the ways in which the nonlinearity management function and the gain/loss coefficient may be used to regulate this nonautonomous rogue wave solution. It has been suggested that by adjusting the nonlinear function and the gain/loss coefficient, it may be possible to "capture" rogue waves. Several possible uses for the rogue wave phenomenon are shown, as well as manipulated, by our method.

Li Li Fajun Yu (2021) The 2+1-dimensional Ablowitz-Ladik (AL) equation's non-autonomous discrete bright-dark soliton solutions (NDBDSSs) are obtained. Here, we examine the resulting 2+1-dimensional NDBDSSs in detail, focusing on their dynamic behaviors and interactions. Herein, we propose two distinct strategies for managing 2+1-dimensional NDBDSSs. In the first approach, the time function has no effect on the phase of the wave, thus we can only influence its propagation in space. Using the second way, we have command of the wave's trajectory in space and time. Both types of management have the potential to generate the various propagation phenomena. New non-autonomous discrete bright soliton solution (NDBSS) and dark soliton solution (NDDSS) shapes, as well as their

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interaction behaviors, are obtained. Analytical consideration is given to the unique phenomena and their potential applications in the electrical and optical domains.

Pooja Thakur (2020) Using a discrete nonlinear Schrödinger equation, we calculate the configurational complexity (CC) of soliton and rogue waves propagating over an Ablowitz-Ladik-Hirota (ALH) waveguide. We demonstrate that CC approaches a changing sequence of global minima over a certain range of the soliton transverse direction propagating along the parametric time. Maximum information compression in the momentum modes through the Ablowitz-Ladik-Hirota waveguide may be seen at these minimums. We compute the CC for rogue waves as a function of the background amplitude modulation and demonstrate that it possesses two crucial characteristics: a maximum at the optimal value for the rogue wave inception (the "gradient catastrophe") and saturation at the point where the rogue wave disperses into its constituent wave modes. We demonstrate that when the discrete rogue wave advances in time, greater levels of modulation amplitude led to saturation at earlier times.

Xiaoyu, wu & Tian (2018) In this study, we explore the Ablowitz-Ladik equation in discrete dimensions (2+1), which is used to explain nonlinear waves in nonlinear optics and Bose-Einstein condensation. Using the Kadomtsev-Petviashvili hierarchy reduction, we are able to determine the Gramian-based solutions for the rogue waves. We visually examine the effects of the focusing coefficient and the coupling strength on the first-, second-, and third-order rogue waves. The rogue wave's crest and the backdrop fade away as the focusing coefficient's value rises. The rogue wave's rise and decay take less time as the coupling strength value rises. The tallest peak of a high-order rogue wave is presented alone, while the smaller humps are shown as triangular and circular patterns.

Coupled Ablowitz-Ladik Systems

Nonlinearities caused by SPM and XPM will be examined in a discrete system. We can accommodate SPM and XPM with a wide range of stokes:

$$i(A_n)_t + \beta(A_{n+1} + A_{n-1} - 2A_n) + \sigma\left(|A_n|^2 - |B_n|^2\right)(A_{n+1} + A_{n-1}) = 0,\ A_n = A_n(t),$$

$$i(B_n)_t - \beta(B_{n+1} + B_{n-1} - 2B_n) - \sigma\left(|A_n|^2 - |B_n|^2\right)(B_{n+1} + B_{n-1}) = 0,\ B_n = B_n(t).$$

We first implement a change of variable:

$$A_n = \phi_n \exp(-2\beta i t),\quad B_n = \psi_n \exp(2\beta i t),$$

to derive

$$i(\phi_n)_t + [\beta + \sigma(|\phi_n|^2 - |\psi_n|^2)](\phi_{n+1} + \phi_{n-1}) = 0,$$

$$i(\psi_n)_t - [\beta + \sigma(|\phi_n|^2 - |\psi_n|^2)](\psi_{n+1} + \psi_{n-1}) = 0.$$

The plane wave or continuous wave is given by

$$\omega_1 = 2[\beta + \sigma(\rho_1^2 - \rho_2^2)]\sin k_1,\quad \omega_2 = -2[\beta + \sigma(\rho_1^2 - \rho_2^2)]\sin k_2.$$

$$\phi_n = i^n \rho_1 \exp[i(k_1 n - \omega_1 t)],\quad \psi_n = i^n \rho_2 \exp[i(k_2 n - \omega_2 t)],$$

Now that the Hirota bilinear transform has been shown to work well for the situation of a single component, it will be used more generally:

$$\phi_n = i^n \frac{G_n}{f_n}\exp[i(k_1 n - \omega_1 t)],\quad \psi_n = i^n \frac{H_n}{f_n}\exp[i(k_2 n - \omega_2 t)].$$

Background plane wave's wavenumber and angular frequency will still be about Eq. but in a more comprehensible form.

$$\omega_1 = -i[\beta + \sigma(\rho_1^2 - \rho_2^2)][\exp(ik_1) - \exp(-ik_1)],$$

$$\omega_2 = i[\beta + \sigma(\rho_1^2 - \rho_2^2)][\exp(ik_2) - \exp(-ik_2)],$$

along with this new restriction

$$\beta + \sigma(\rho_1^2 - \rho_2^2) = -1.$$

Hence, we may write down the bilinear form as

$$D_t G_n \cdot f_n = (G_{n+1} f_{n-1} - G_n f_n)\exp(ik_1) + (G_n f_n - G_{n-1} f_{n+1})\exp(-ik_1),$$

$$D_t H_n \cdot f_n = (H_n f_n - H_{n+1} f_{n-1})\exp(ik_2) + (H_{n-1} f_{n+1} - H_n f_n)\exp(-ik_2),$$

$$f_{n+1} f_{n-1} + \beta f_n^2 + \sigma(|G_n|^2 - |H_n|^2) = 0 .$$

SOME PROPERTIES OF NONLINEAR SCHR¨ODINGER EQUATIONS

$$\mathrm{i}\Psi_t = \alpha_1 \Psi + \alpha_2 \Psi_{xx} + \alpha_3 \Psi |\Psi|^2 + \alpha_4 [$$

It was developed as a continuum approximation to a discrete equation with nonlinear coupling, where the subscripts now indicate partial derivatives. In paper II, we will utilize this equation as a case study to derive characteristics of the continuum nonlinear Schr odinger equations. Eq. (3.1) is a good candidate for a Lagrangian formulation, so we begin by noting that it can be derived from first principles. What we mean is that a Lagrangian exists.

$$\mathsf{L} = \int \mathcal{L}\, dx$$

A Lagrangian density L, which is a functional of the variables, and, where the equation is a fixed point of the action integral.

$$\mathsf{S} = \int \mathsf{L}\, dt$$

Thus, the variation's Euler-Lagrange equations should yield Eq (3.1). Due to the complex nature of the field variables, the equation forms a system of two connected real equations, one each in the real and imaginary sections of the complex variables. Then, S is independently varied with regard to u and v, two real variables, leading to two coupled equations.

$$\frac{\partial S}{\partial u} = \frac{\partial S}{\partial \Psi} + \frac{\partial S}{\partial \Psi^*} = 0, \qquad \frac{\partial S}{\partial v} = i\left(\frac{\partial S}{\partial \Psi} - \frac{\partial S}{\partial \Psi^*}\right) = 0.$$

We may alternatively calculate independent variations of S with respect to and I using these equations. In mathematics, the Euler-Lagrange equations

$$\frac{\partial S}{\partial \Psi} = 0 \iff \frac{\partial \mathcal{L}}{\partial \Psi} - \frac{\partial}{\partial t}\frac{\partial \mathcal{L}}{\partial \Psi_t} - \frac{\partial}{\partial x}\frac{\partial \mathcal{L}}{\partial \Psi_x} = 0$$

$$\frac{\partial S}{\partial \Psi^*} = 0 \iff \frac{\partial \mathcal{L}}{\partial \Psi^*} - \frac{\partial}{\partial t}\frac{\partial \mathcal{L}}{\partial \Psi^*_t} - \frac{\partial}{\partial x}\frac{\partial \mathcal{L}}{\partial \Psi^*_x} = 0$$

We have recently found a Lagrangian density that produces results. The fact that Eq. is essentially the complex conjugate of Eq. demonstrates that the Lagrangian density must be symmetric with respect to an exchange of the variables and I if L is real (3.5b). Hence, it is possible to demonstrate that the Lagrangian density is

$$\mathcal{L} = \frac{\mathrm{i}}{2}\Psi^* \Psi_t - \frac{\alpha_1}{2}|\Psi|^2 + \frac{\alpha_2}{2}|\Psi_x|^2 - \frac{\alpha_3}{4}|\Psi|^4 + \alpha_4(\Psi$$

SYMMETRIES AND CONSERVED QUANTITIES

One particularly nice feature of the Lagrangian formulation is that it allows us to establish connections between certain aspects of symmetry and variables that are preserved in the dynamics described by the equation. The conserved values are helpful for regulating the correctness and validity of numerical simulations, and they also provide crucial information on the behavior and features of the system. E. Noether was the first to formally express the connection between symmetries and conserved values for Lagrangian systems in a theorem. We propose the tiny parameter to restrict the scope of the theorem to continuous symmetries that have an expression in terms of infinitesimal generators. Let's pretend now that the shape has been transformed

$$t \mapsto t + \epsilon T.$$

$$x \mapsto x + \epsilon X$$

$$\Psi \mapsto \Psi + \epsilon \Upsilon$$

with T, X, and Y functions of t, x,, t, and x, without changing the action integral S. Hence, if we assume that

$$\frac{d\mathcal{I}}{dt} + \frac{d\mathcal{J}}{dx} = 0$$

As mentioned before, if the current density J tends to a constant value at infinity or if periodic boundary conditions are used, then I = I dx is a conserved quantity, or constant of motion, as determined by the integration of Eq. over x. The theorem is proven by deriving the continuity equation explicitly from the transformation, and its application to nonlinear Schr odinger equations is discussed. Any continuous symmetry that can be expressed in the form given by Eq. will result in a conserved quantity, which is the major consequence of the theorem. Other symmetries, such as symmetry under inversion of time or a spatial coordinate, are not discussed. Also, any conserved quantity may not have a corresponding symmetry (the inverse of the theorem). Integrable equations are the most clear-cut examples; these include the cubic NLS equation and others with an infinite number of conserved variables but seemingly just a few continuous symmetries, such translation in time or space. These later symmetries are the most basic sort of symmetry on Eq. (3.7), and they hold true whenever the Lagrangian does not rely directly on the variables in question.

Starting with the invariance under temporal translation, we now construct several conserved values for the continuum equation. T = 1 and X = Y = 0 indicate an infinitesimal transformation, which is what we use in Eq. The density of the Hamiltonian may be defined by plugging in a new term in Eq.

It seems intuitive to associate H, the Hamiltonian we get by considering temporal translations, with the system's overall "energy," but keep in mind that there is no assurance that H is tied to any physical energy in a real-world application. The Hamiltonian flux density is the current density connected to H, and it is found in Eq. (3.10).

$$J_{(\mathcal{H})} = 2\,\mathrm{Re}\left\{\Psi_t^* \left[\alpha_2 \Psi_x + 2\alpha_4 \left(\Psi^2 \Psi_x^* + 2|\Psi|^2 \Psi_x\right)\right]\right\}$$

This will characterize the movement of "energy" across the system. The Hamilton equations of motion may be written as a combination to provide and its complex conjugate.

$$\mathrm{i}\Psi_t = \frac{\partial \mathcal{H}}{\partial \Psi^*} - \frac{d}{dx}\frac{\partial \mathcal{H}}{\partial \Psi_x^*}$$

Eq. (3.7) is likewise translationally invariant, with X = 1 and T = Y = 0, and the equivalent conserved quantity, P = P dx, is intuitively called momentum.

TWO-DIMENSIONAL ARRAY OF SQUARE WAVEGUIDES

Analytical treatment is not as straightforward for the two-dimensional array shown in Fig. A.1b. An analytical solution for a rectangular dielectric waveguide has not yet been found. The sections that share a boundary with the interior region are the only ones in the exterior region where the approximate solution for modes that are not close to cut-off is valid; no expressions are given for the fields in the sections in the diagonal directions where the fields are assumed to be weak. Although this solution only accounts for the interaction between adjacent waveguides, it is still possible to draw the conclusion that the relative sizes of the nonlinear coupling parameters to the on-site nonlinearity are about the same as for the array of slab waveguides. Furthermore, connection between adjacent elements in the two-dimensional array is possible along the diagonal. This approximate solution cannot be utilized to assess the strength of this linear connection. Even though this solution does not meet all the needed boundary conditions and leaves out certain field components, we can still construct an order of magnitude estimate by choosing expressions for the fields that reflect the key behavior in the distinct areas. Given that the electric field's dominant transverse field component is assumed to be, the fields must oscillate in the inside and diminish exponentially in the outside.

$$\mathcal{E}(x,y)=\begin{cases} A\cos(kx)\cos(ky) & -d\le x,y\le d,\\ A\cos(kd)\cos(ky)\mathrm{e}^{-\tilde{k}(x-d)} & x\ge d,-d\le y\le d\\ A\cos^2(kd)\mathrm{e}^{-\tilde{k}(x-d)}\mathrm{e}^{-\tilde{k}(y-d)} & x,y\ge d.\end{cases}$$

The wave counts for the lowest mode are same in both directions since the waveguides are square. For our purposes, the formulas presented in Eq. (A.9) are sufficient, but they may be simply applied to other areas. In a square waveguide, the mode is degenerate, but in an ideal case, the two orthogonal polarization orientations will not interact with one another. In an isotropic medium, there is no need to discriminate between the modes since they all have the same propagation constant.

CONCLUSION

The discrete A-L equation has been solved to the second rational order. An array of closely connected optical waveguides 14 may be approximated by this solution for errant light waves. Depending on the system's settings, the light may be focused into intense peaks. An analogy between rogue waves in the continuous NLSE 30 and higher-order rational solutions in the discrete A-L equation is our key finding here. Different coefficients and scaling factors were identified in the formulas for the rational solutions in the discrete case compared to the NLSE. The center maximum of a discrete rogue wave is often substantially taller than in the analogous NLSE scenario, which is the most striking characteristic of a rogue wave. Furthermore, we have provided rational solutions for the more general discrete Hirota equation, which includes the discrete A-L equation and the discrete mKdV equation as special instances.

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