Validation of Hertzian Theory for Contact Stress Computation Using FEM

Swati P. Gundewar[1], Syed Aalam[2]

[1 2]*Department of Mechanical Engineering, Matoshri Pratishthan Group of Institutions, Nanded Maharashtra 431606*

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Abstract –This paper represents the study on validation Hertz contact stress statement for two elastic solids in contact. The hertz theory states that when the load is applied on two bodies are in contact, which results in the formation of very high pressure at the point of contact, this pressure induces elastic deformation at the point of contact. This elastic deformation leads to accountable high stress at the point of contact which may cause failure in structure [5]. In this research, contact stresses were analyzed for two semicircular disks as point contact and two semi cylinders as a line contact. Validation of the result was performed by both the analytical method and the numerical method. Post validation application of hertz theory for the nonlinear contact like mating pair gear during rotation was studied in this research. Contact stress was calculated between mating spur gear.

Key Words: *contact stresses, semi-circular disks, spur gear, validation, point contact, line contact*

I. INTRODUCTION

During the past few years, significant work has been done in the theoretical study of stress and strength conditions arising when the two solid bodies are together in contact. This gives us to increase interest to study all machine members which are in contact to work under the same conditions [1]. Example of such a machine is gear and roller bearings. When two elastic bodies are in contact, there is a formation of very high pressure. This may cause failure in design if not considered during the design phase of equipment [4]. Hertz Theory defines to calculate the contact area and pressure between the two surfaces and predicts the resulting compression and deformation induced in the solids. There are two types of contact i) confirming contact -The contact between two surfaces fit exactly together without deformation is called confirming the contact. These contacts are ideal contact and it is never be used in engineering calculation. ii) Nonconforming contact- the contact between two surfaces deform when they are in contact is called non-conforming contact.

When two non-conforming solids are in contact, there would be a single point or line contact. Under the application of a small amount of load would result in inelastic deformation at the point of contact, which may cause failure in design. The area of contact would be finite enough compared with the dimensions of the two bodies. This theory predicts the shape of the contact area and its penetration at the point of contact [5]. It can determine its limiting size of the axes of the elliptical contact area and the relative displacement of bodies.

The Contact stress is induced in solid bodies when two surfaces of elastic bodies are pressed together by external load causes considerable stresses, that stresses on the surface of contact are the major cause of design failure. There are some examples around us are having contact stresses role significant in the area of contact [2]

- Stresses between the locomotive and the railroad rail
- Stresses between teeth of pair of the mating gear
- Stresses between Cam and valve of an engine

The above examples define contact stresses are cyclic and induced a repeated number of times, this results in fatigue failure of the component. There would be a starting of localized fracture due to stresses at the vicinity of contact between two surfaces. The fact that contact stresses continuously leads to fatigue failure, this explains why these stresses limit the load-carrying capacity of the member in contact.

This paper validates the statement of Hertzian theory of contact stress by comparing analytical and numerical value for both points as well as line contact

II. POINT CONTACT

GEOMETRY--Two semi-circular disks made up of elastic material are pressed together by an external force of P, as shown in figure 1. These two bodies are initially in point contact. The section of the boundaries of two bodies at the vicinity of contact is smooth enough before the application of load. Let's consider principal radii of curvature of upper solid at the point of contact R1 and R1'. Similarly, R2 and R2' are principal radii of curvature of the lower solid. Line of action of load P is perpendicular to a plane that is tangent to both bodies at the point of contact. From figure1 there is no tendency for one body to slide concerning another body.

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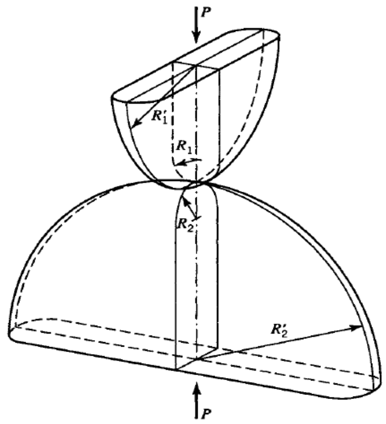

Two curved surfaces of different radii pressed against each other.

Fig-1: Two Curved surfaces of solids

III. PROBLEM DEFINITION

To evaluate the effectiveness of the proposed Hertz theory of contact using a theoretical solved problem [2]. The two semicircular disks made up of steel material are in contact under an applied force of 4500N. As shown in Table no 1

Table -1: Properties of semicircular disk

Poisson's ratio	0.29	R1 (mm)	60
Elasticity (Gpa)	200	R1' (mm)	130
Clearance (mm)	0.00001	R2 (mm)	80
ά	0	R2' (mm)	200

The set of following equitation's used for the calculation of contact stress at the point of

$$B = \frac{1}{4}\left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_1'}+\frac{1}{R_2'}\right) + \frac{1}{4}\sqrt{\left[\left(\frac{1}{R_1}-\frac{1}{R_1'}\right)+\left(\frac{1}{R_2}-\frac{1}{R_2'}\right)\right]^2 - 4\left(\frac{1}{R_1}-\frac{1}{R_1'}\right)\left(\frac{1}{R_2}-\frac{1}{R_2'}\right)\sin^2\alpha}$$

$$A = \frac{1}{4}\left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_1'}+\frac{1}{R_2'}\right) - \frac{1}{4}\sqrt{\left[\left(\frac{1}{R_1}-\frac{1}{R_1'}\right)+\left(\frac{1}{R_2}-\frac{1}{R_2'}\right)\right]^2 - 4\left(\frac{1}{R_1}-\frac{1}{R_1'}\right)\left(\frac{1}{R_2}-\frac{1}{R_2'}\right)\sin^2\alpha}$$

$\boldsymbol{B = 0.01458mm^{-1}, A = 0.0063454mm^{-1}}$

$$\Delta = \frac{2(1-\nu^2)}{(A+B)E}$$

Δ = **437.70 x 10-6**

B/A = **2.29**

The coefficients needed to calculate b, σ, δ. From the graph of Stress and deflection coefficients for two bodies in contact at a point for the ratio of B/A = 2.29, Cb =0.65, Cσ =0.97 ,Cδ = 1.5 $b = c_b \sqrt[3]{P\Delta}$

b **= 0.777 mm**

b/Δ **=1775.41 MPa**

$\sigma_{max} = - C_\sigma * b/\Delta$

σ_{max}= **- 1722 MPa**

$$\delta = c_\delta \frac{P}{\pi}\left(\frac{A+B}{\frac{b}{\Delta}}\right)$$

δ = **0.0253 mm**

IV. CONSTRUCTION OF FINITE ELEMENT MODEL

Hertzian theory of contact stress was validated by comparing the contact stresses obtained from the analytical method, with stresses observed in the finite element method (i.e. numerical solution).

The dimension of the semicircular disc was referred from the problem defined in the book of Advance Mechanics of Manufacturing, Arthur P Boresi [2]. The 3D CAD model was prepared by considering the coordinates of the semicircular disc. The modeling was completed in ANSYS APDL17.5 GUI as shown in Fig 2. The discretization of two solid bodies was accomplished by assigning tetrahedral mesh. The total mesh count was 8185 nodes. The quality of the mesh was compared with the standard meshing quality of the solid component. The obtained tetrahedral mesh having good quality. The discretized model of two semicircular discs is shown in fig 3.

A. Material

As mentioned in the problem material used for two solids in contact is structural steel [2].

a) Mechanical properties of steel:

- Density = 7800 Kg/m3
- Elastic modulus = 200GPa
- Poisson's ratio = 0.29

B. Modeling

There are two approaches to model 3D CAD design in ANSYS. The design can be imported from external CAD design software or it can be modeled in ANSYS software itself. It is easy to model standard shapes and sizes in ANSYS. The modeling is ANSYS is always preferred on importing it from external third-party software to avoid an error during format change of different CAD files. The modeling of two semicircular solid of standard shape was achieved in ANSYS APDL 17.5. The Modeled 3D CAD design of two semi-spherical solids are shown in Fig 2.

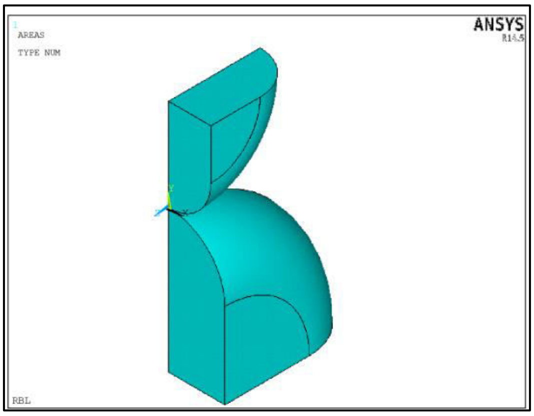

Fig.-2: 3D design of two solids

C. Contact Setup

The contact between two semicircular surfaces is points contact. It can be created by selecting a node in the contact area. The bonded type of contact is used for contact between two solids. The point to surface contact was assigned by contact element CONTA175. The target nodes were assigned by the target element TARGE170.

a). Meshing

Meshing is sensitive towards the solution point of view. The fine mesh was provided at the contact area as it is an area of interest for us. For the meshing of contact surfaces, the nodes of contacting surfaces were selected separately and meshed using small element size. The semicircular disk was meshed by solid element 187 (20 node tetra- element), Increase in nodes to be selected leads to the more accurate solution but require more time and space in system.

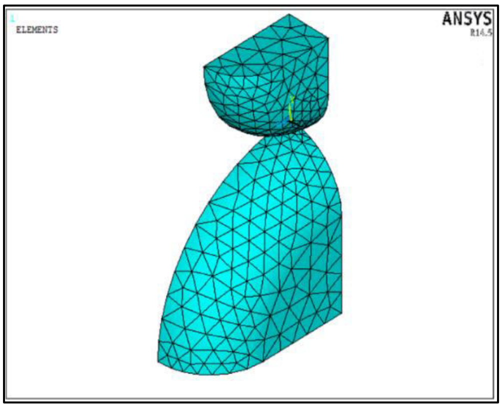

Fig-3: Meshing

D. Analysis Setup

The quarter part of the semicircular solid disc was considered in analysis; hence the symmetric boundaries have to be used at the external surface of solid. The boundary condition and constrain of this solid include symmetric surfaces, fixed surfaces and externally applied the load at the tangent plane of the solid disc. The solution for this analysis was calculated using ANSYS mechanical solver.

a) Constraints

As per the aim of this project, to calculate contact stresses at point of the contact region. We need to focus on the contact area of solids as a region of interest. The lower solid disk is fixed in the Y direction (vertical displacement constrained) by fixing its bottom surface. The upper disk can translate in the vertical direction (i.e. vertical displacement is free for upper disk). The maximum compressive load of 4500 N was applied at the center of the disk as shown in fig 4. The coupled boundary condition was applied on the top surface of the upper disk so that all nodes would have similar behavior against applied load [5].

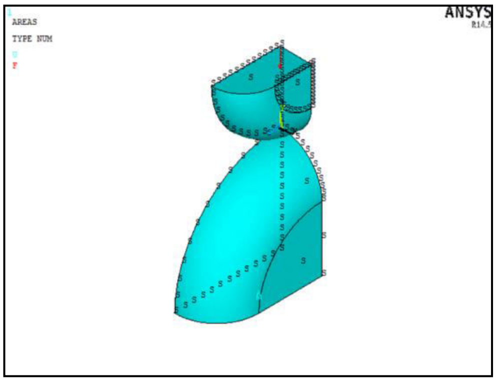

Fig-4: Boundary condition

E. Result Contact Stress Analysis

The maximum Von-Mises stress of 845 MPa was observed vicinity of the point of contact as shown in Fig 5.

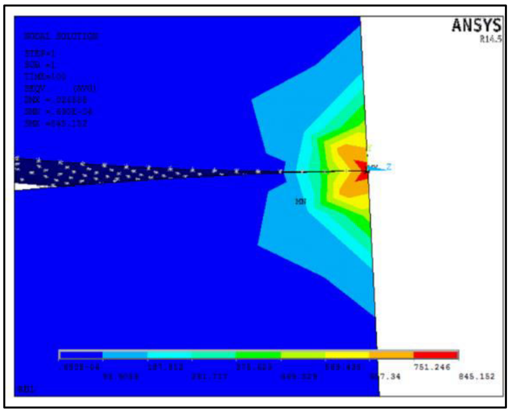

Fig-5: Volume: Von Mises Stress (MPa)

The maximum displacement was0.248 mm was observed on the upper disk as it is free to translate in a vertical direction as shown in Fig 6.

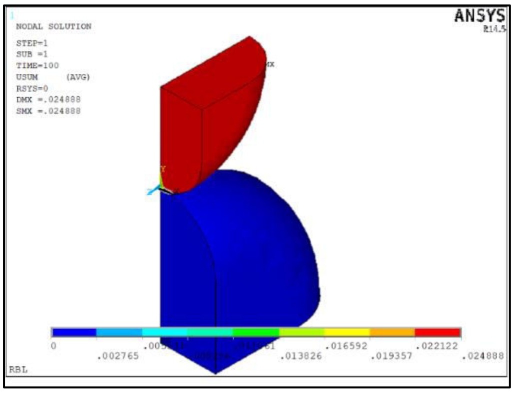

Fig-6: Displacement field (Material)

A. Validation of result

The result of the analysis was compared with the analytical calculation represented in Table 2

From the results, it is clear that the value stresses are approximately equal. The variation in stress results was less than 8 %, hence it can be concluded that the results obtained from ANSYS (Numerical method) are approximately equal to analytical calculated results. As per the standard validation procedure, the variation in results up to 10 % is acceptable. To minimize the error in the results, refine mesh is necessary at the point of contact. After refining mesh obtained results are given in the table

Table-2: Results comparison

	Approach	Max PrincipalStress
Theory	0.0253mm	1722 N/mm^2
ANSYS coarse Mesh	0.0248mm	1598N/mm^2
ANSYS FineMesh	0.0256mm	1765 N/mm^2

The new value obtained for the approach is nearly the same as theoretical value, also maximum principal stress for finer mesh shows the variation of less than 3% concerning the theoretical value. This shows that even both meshes provide a unique solution, the finer mesh generates a more accurate result.

V. LINE CONTACT

Two parallel linear elastic half-cylinders are having radius R forced by small distributed pressure P. The region of contact approximately equal to straight-line element. The line Contact between two circular cylinders is shown in figure5. The base of the lower cylinder is fixed in all directions, to study frictionless line contact between two semi-circular cylinders.

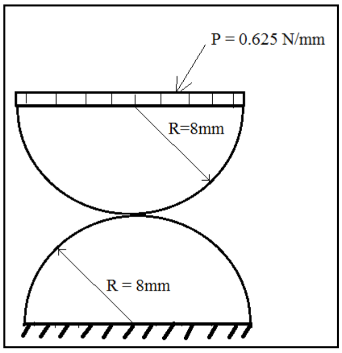

Fig-5: Modelling of half cylinder

VI. PROBLEM DEFINITION

To calculate contact stress between two parallel linear elastic cylinders of having radius 8mm are forced by the pressure 0.625N/mm, in other words, the application of force is 10N. All design parameters are given in the below table

Table-4: Design Parameter

$$\sigma_2 = \sigma_y = -p_{\max}\left[\left(2-\left(\frac{z^2}{b^2}+1\right)^{-1}\right)\sqrt{\frac{z^2}{b^2}+1}-2\left|\frac{z}{b}\right|\right]$$

N	
F	
Radius	8mm
Load intensity (F)	0.625 N/mm

The set of following equitation's used for the calculation of contact stress at the point of contact between two cylinders:

b = half-width of the rectangular contact area
P_{max} = maximum contact pressure along the centreline
σ = maximum contact stresses

$$b = \sqrt{\frac{4F\left[\frac{1-\nu_1^2}{E_1}+\frac{1-\nu_2^2}{E_2}\right]}{\pi L\left(\frac{1}{R_1}+\frac{1}{R_2}\right)}}$$

b = 0.170 mm

$$p_{\max} = \frac{2F}{\pi b L}$$

P_{max} = 2.34 N/mm^2

σ_2 = 5.25N/mm^2

VII. METHODOLOGY AND IMPLEMENTATION

The objective of this study is to calculate contact stress between point contact, line contact, and sliding contact and compare the results with analytical calculation. The design of the machining component is inadequate, due to high stresses generated at these small vicinity regions of contacting surface, hence the study shows the use of the numerical method to calculate contact stress will help to predict design failure [6].

The method of modeling two-cylinder is nearly the same as modeling two semicircular spheres. The 3D CAD model is prepared in ANSYS APDL.

A. Material

The material used for two solid cylinders in contact is structural steel [7].

a) Mechanical properties of steel:

- Density = 7800 Kg/m^3
- Elastic modulus = 200GPa
- Poisson's ratio = 0.29

It is assumed that contact condition at the point of contact of two semicircular cylinders is frictionless as shown in figure 7.

B. Modeling

The line contact between cylinder bodies was modeled by considering two half-cylinders. The dimension of the cylinder is derived from the problem definition. The modeling was completed in ANSYS APDL as shown in Fig 6.

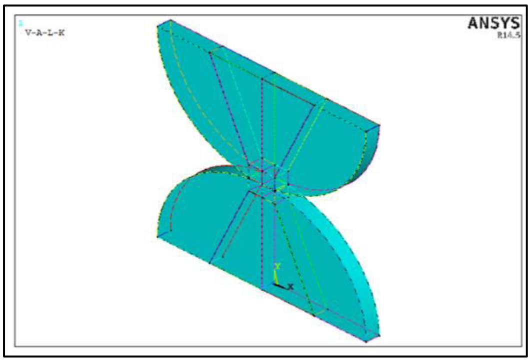

Fig-6: Modelling of half cylinder

C. Meshing

The contact area between the two cylinders is the focus of study hence geometry of contact surfaces needs to be fine [6]. The fine-meshed element was generated near the point of the contact area. Finer meshing leads to accurate results but requires more time and high system resources. It is recommended that not to have fine mesh everywhere in the model to reduce computation time as well as system space. The mesh on the cylinder is shown in figure 7.

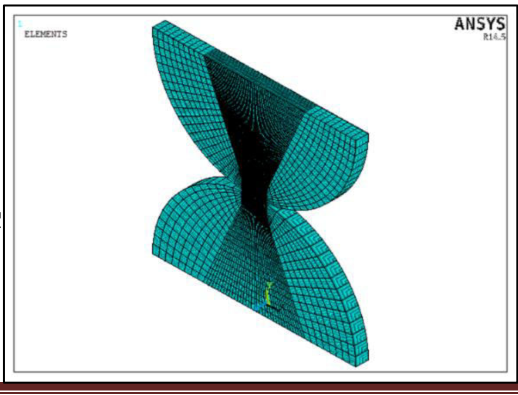

Fig-7: Meshing

D. Constraints

The bottom of the lower cylinder is fixed in all directions. The simulation uses one load step that is top of the upper cylinder is ramped by uniform pressure 0.625N/mm. The externally applied pressure pushes the top cylinder so that it will induce stress in the contact region.

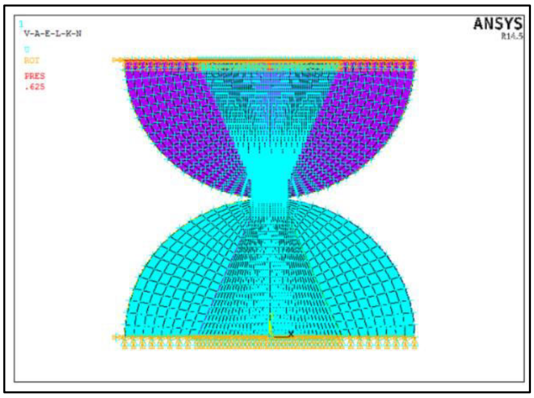

Fig-9: Boundary condition

E. Nodal Solutions

The normal contact stresses along the contact surfaces are shown in the above figure 10. The maximum stress observed at the point of contact is 5.5 MPa. The total deformation of 0.186 mm was observed on the upper cylinder as shown in Fig 9.

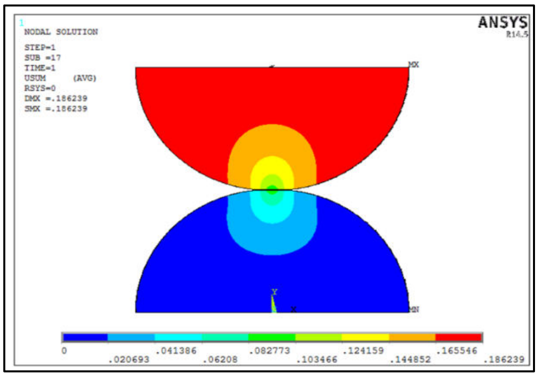

Fig-9: Deformation

F. Nodal von misses stresses:

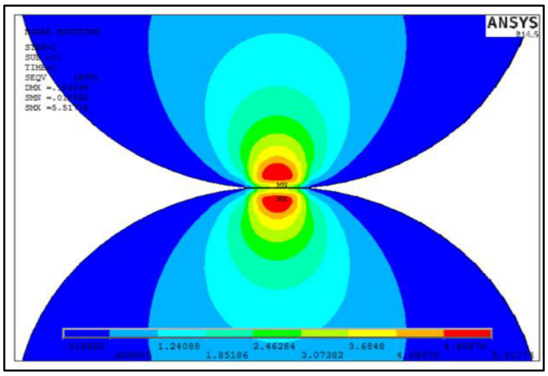

Fig-10: Contact Stress at point of contact

G. Validation of result

When we compare the result observed by numerical method (FEM) with theoretical values, the value of maximum stress is nearly equal as shown in Table 3

Table-3: Results (Line contact analysis)

	Approach	Max Principal Stress
Theory	0.170mm	5.25 N/mm^2
ANSYS	0.186mm	5.5 N/mm^2

The difference in the result obtained is less than 8% and it is acceptable. Hence, we can conclude that the stresses observed in the analytical calculation are realistic stress and it is well validated using numerical method (ANSYS). The Hertzian theory of contact stress is validated using both point contact and line contact. The application of this study is used in the design of a machine component where two solid bodies are in contact. The most common application of Hertzian theory is a study of contact stresses induced in gear teeth. Hence the study of gear contact is presented in this paper.

VII. GEAR CONTACT

Gear is useful for changing the speed and direction of a power source. Gears mainly used as a mechanical element of power transmission. Generally, in gear transmission, sudden load changes from the viewpoint of transmission [9]. In other words, the load acting on teeth depends on the meshing stiffness of that pair. This will increase a variation in the load distribution across contact points.Stress analysis for gear teeth is a limiting factor for designers. Stress analysis mainly focuses on the determination of the region of stress concentration where the fracture occurs [8].

As the numerical method is the best approach to calculate contact stress at mating solid, hence we used the FEM method to calculate contact stress at mating gear. There are two methods for analyzing the stress concentration between meshing gear teeth in the finite element method. One is a direct application of load at the single gear teeth, and observe bending stresses of gear. However, to evaluate contact stresses between gears, the second method has to be used that is using mating of gear tooth profile. The meshing of the gear pair will provide realistic stress at the contact region [9]. The present study uses a two-dimensional FEM model for calculating contact stress between the gear tooth profile. The mating of gears is modeled in ANSYS WORKBENCH software as shown in Figure -1.

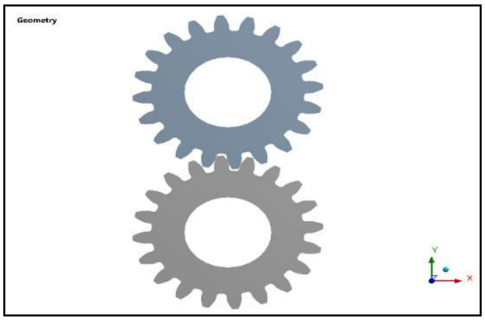

Fig-10: Geometry of gear

1. Meshing

The fine meshing at the contact of mating gear is important during meshing. The discretization of two-dimensional geometry with shell element Shell181 was accomplished in ANYS Mechanical. The size of the mesh is controlled by proving edge sizing at teeth of a gear. The adequately small size of elements was given at the point of contact. The meshing of mating gear is shown in Fig 11.

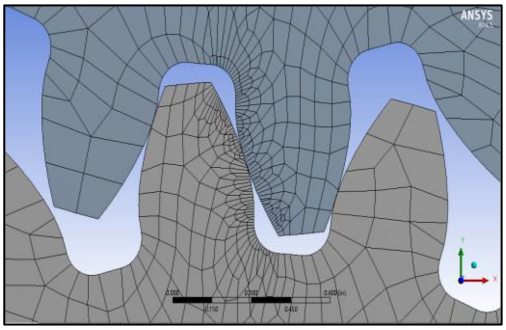

Fig-11: Meshing of gear

2. Constraints

During constraining, lower gear is fixed. So that there is no rotation of the lower gear. The moment of 15000 lbs-in was applied on upper gear. The contact between mating gear is frictionless, as it will allow to slide and separate it from the contact region. The boundary condition for mating gear is shown in Fig 12.

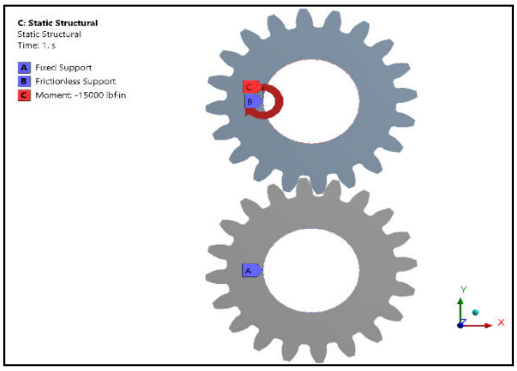

Fig 12 – Boundary Condition

3. Solution:

The maximum stress of 2.1 Psi was observed at the contact of the mating gear. It is a real stress and it has to be taken into account during the design of the gear, as it affects the root of gear if the value of contact stress is inflated to its failure limit. This method of calculating contact stress can be used for all types of gears where line or point contact is a focus of study. The result shows contact stresses are less than the failure stress, hence the design of contact stress is adequate for applied loading. The contact stress at mating spur gear is shown in fig -11.

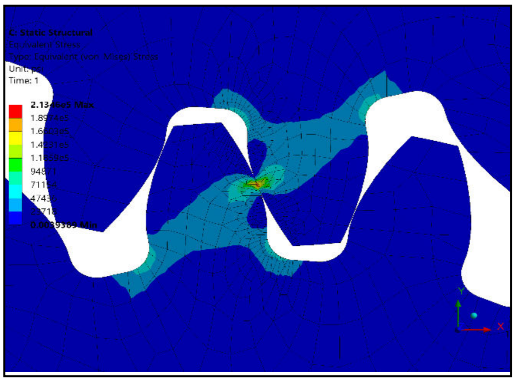

Fig-11: Result- Contact stress at mating gears

VIII. CONCLUSION

This study represents the validation of hertz theory to calculate the contact area and pressure between the two solid bodies and predict the resulting compression and stress-induced in the objects.

Hertz's theory of contact is important in mechanical design. This theory allows you to predict contact area, pressure, compression and stresses for continuous non-conforming contact under loading condition. The small contact area that results causes considerable design difficulties, may require an alternative design method.The obtained solution indicates that the change in contact stresses of different contacting with contact positions.

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