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Vibration Analysis of Composite plate with Central rectangular cut-out using FEM

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Abstract

Plate structures are important engineering substructures, especially where weight is a major consideration. Thin plate structures with arbitrary cutouts are unavoidable due to the high demand for laminates in a range of engineering applications. The existence of cutouts will alter the frequency of free vibration. FEM was used to investigate the natural frequencies of isotropic and composite laminated plates, as well as the effects of various plate characteristics with and without cut-out. A nine-node iso-parametric element is used in the numerical analysis. The impact of the number of layers, angle of fibre orientation, width to thickness ratio, modulus ratio, and cutout size on plate natural frequencies is investigated. For various values, the nondimensional fundamental frequency of vibration is displayed.

Key Words: Finite Element Method, laminate composite plate, lumped mass, cut-out.

1.INTRODUCTION

Plates are subjected to transverse load conditions, which result in transverse deflections. Bending and shear action work together to support transverse loads. The smallest side of a thin plate is greater than 20 times the thickness, and the displacements in the X, Y, and Z directions are minimal in comparison to the thickness. The middle surface of a thin plate remains unstrained during bending, and the normal to the middle surface before deformation remains normal to the same after deformation, with little rotating inertia. The original body or structure is modelled in FEM as an assemblage of pieces joined by a finite number of joints known as 'Nodes' or 'Nodal Points.' To create a solution for the overall body or structure, the attributes of the elements are formulated and integrated. Simple functions known as'shape functions' are chosen to approximate the variation in displacement within an element in terms of the displacement at the nodes of the element in the displacement formulation extensively used in finite element analysis. This is similar to the Rayleigh-Ritz functional approximation method, with the exception that the approximation to the field variable is done at the element level. The qualities of a composite material are usually derived from its constituents. Composite materials such as plywood and reinforced concrete have been utilised for a long time. Layers of diverse characteristics are bound together to operate as an essential portion of laminated composite materials. Particles of various materials are kept together in a matrix in particulate composite materials. Nowadays, fibre reinforced plastics are being increasingly used in aerospace applications due to their high specific strength, high specific stiffness and low density. In addition, they have good corrosion resistance. A designer can easily tailor these materials for different applications. In fibre reinforced plastic composites, first a thin lamina is prepared from fibres and matrix (sometimes a lamina may also be made of woven fabric). Lamina with different fibre orientations is bonded together to form an integral structural component, which is known as laminate. A lamina is considered to be homogeneous at microscopic level. Its stress-strain behaviour is commonly referred to as linear elastic. Laminates can be symmetrical, anti-symmetrical, or asymmetric. Researchers have proposed a number of viable theories/formulations to address the shortcomings of composite structures and advanced structural materials compared to traditional materials. To fill the gap, a number of studies have looked at the static and vibration responses of laminated composite plates.

2. Equation of Motion

In this study, the finite element method has been used for calculating the free vibration of plates. The middle plane of the plate of the element is considered as the reference plane. The element used in this present work is nine-node iso-parametric finite element with 5 DOF per node (u, v, w). The independent field variables are u, v and w, where w is the transverse displacement while u and v are the corresponding in-plane displacements and are the total rotations in bending. The interpolation functions used for the representation of element geometry. The displacement field at a point within the element in terms of nodal variables are

u = $\sum_{r=1}^{9} N_r u_r$, v = $\sum_{r=1}^{9} N_r v_r$, w = $\sum_{r=1}^{9} N_r w_r$,
$\theta_x = \sum_{r=1}^{9} N_r \theta_{x_r}$, $\theta_y = \sum_{r=1}^{9} N_r \theta_{y_r}$

The relationship between strains and stresses are derived from Hooke's law by, [σ]=[D]{ε}

The generalized stress vector {σ} in the above equation is,

$\{\sigma\}^T = [N_x\ N_y\ N_z\ M_x\ M_y\ M_{xy}\ Q_x\ Q_y]$

While the generalized strain vector {ε} may be written in terms of displacement field as,

$$\{\varepsilon\}^T = \left\{\frac{\partial u}{\partial x}\ \frac{\partial v}{\partial y}\ \left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)\ -\frac{\partial \theta_x}{\partial x}\ \ -\frac{\partial \theta_y}{\partial y} - \left(\frac{\partial \theta_x}{\partial y}+\frac{\partial \theta_y}{\partial x}\right)\ \left(\frac{\partial w}{\partial x} - \theta_x\right)\ \left(\frac{\partial w}{\partial y} - \theta_y\right)\right\}$$

Now the rigidity matrix [D] is written as,

$$[D] = \begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} & 0 & 0 \\ A_{12} & A_{22} & A_{26} & B_{12} & B_{22} & B_{26} & 0 & 0 \\ A_{16} & A_{26} & A_{66} & B_{16} & B_{26} & B_{66} & 0 & 0 \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} & 0 & 0 \\ B_{12} & B_{22} & B_{26} & D_{12} & D_{22} & D_{26} & 0 & 0 \\ B_{16} & B_{26} & B_{66} & D_{16} & D_{26} & D_{66} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & A_{55} & A_{54} \\ 0 & 0 & 0 & 0 & 0 & 0 & A_{45} & A_{44} \end{bmatrix}$$

where,

$$A_{ij} = \sum_{k=1}^{n} (Q_{ij})_k (h_k - h_{k-1})$$

$$B_{ij} = \frac{1}{2} \sum_{k=1}^{n} (Q_{ij})_k ({h^2}_k - h_{k-1}^2)$$

$$D_{ij} = \frac{1}{3} \sum_{k=1}^{n} (Q_{ij})_k ({h^3}_k - {h^3}_{k-1})$$

The generalized displacement within the element in terms of nodal displacement can be expressed as,

{ε}= [B]{d}

The displacement gradient can be related to the nodal displacement in the FEM as,

$$\{\boldsymbol{\varepsilon}\} = \sum_{r=1}^{9} [\boldsymbol{B}] \{\boldsymbol{\delta}\}$$

Where, [B] is the strain matrix containing interpolation functions and their derivatives and {δ} is the nodal displacement vector having order 45x45.

Once the matrices [B] and [D] are obtained, the stiffness matrix of the plate element [K] can be easily derived by the virtual work method and it may be expressed as,

$$[K]_e = \int_{-1}^{+1} \int_{-1}^{+1} [B]^T [D][B] dydx$$

The above expression in local coordinate is written as,

$$[K]_e = \int_{-1}^{+1} \int_{-1}^{+1} [B]^T [D][B] |J| d\eta d\xi$$

The integration has been carried out numerically by following Gauss quadrature technique. In the similar manner, the consistent mass matrix of an element can be derived
and it may be expressed as,

$$[M] = \rho h \int_{-1}^{+1} \int_{-1}^{+1} \left([N_u]^T [N_u] + [N_v]^T [N_v] + [N_w]^T [N_w] + \frac{h^2}{12} [N_{\theta_x}]^T [N_{\theta_x}] + \frac{h^2}{12} [N_{\theta_y}]^T [N_{\theta_y}] \right)$$

The element stiffness matrix and mass matrix having an order of forty five are evaluated for all the elements and they are assembled together to form the overall stiffness matrix $[K_0]$ and mass matrix $[M_0]$. Once $[K_0]$ and $[M_0]$ are obtained the equations of motion of the plate may be expressed as,

$[K_0] = \omega^2 [M_0]$

After incorporating the boundary conditions in the above equation it is solved by the simultaneous iterative technique to get frequency ω for first six modes.

3. BOUNDARY CONDITION

The purpose of boundary condition in any solution is to avoid the rigid body motion and to get the responses by reducing the number of field variables. In order to solve the governing equations as discussed in the above-mentioned line are solved using different support conditions. The supports conditions are discussed mathematically and a schematic presentation of a plate have been given in Figure 1.

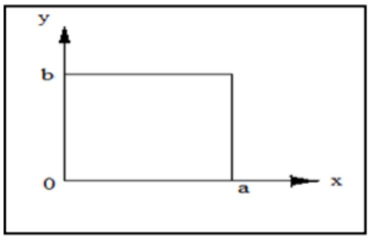

Figure 1: Schematic presentation of plate

Clamped on all edges:

u= v = w = = 0, at x=0, a and y =0, b.

Simply supported edges

v = w = = 0, at x = 0, a;

u = w = = 0, at y = 0, b.

4. RESULTS AND DISCUSSIONS

In this work, the frequencies of isotropic and composite plate with cut-out are analyzed under different situations, which include different boundary condition, thickness ratio (h/a), cut-out ratio (c/a).

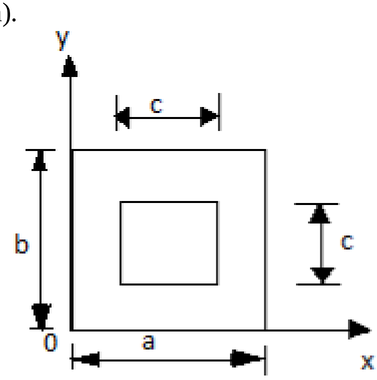

Figure 2: Plate geometry with cut-out.

A. Isotropic plate with cut-out.

A square isotropic plate with a central rectangular cut-out is considered. The plate has thickness ratio 0.01, Poisson's ratio (ʋ) 0.3, Young's modulus 10.92x106 N/m^2, and shear modulus 4.2x106 N/m^2. Non-dimensional frequency parameter is

$$\lambda = \omega a^2 \sqrt{\rho h / D} .$$

Table -1: Comparison of frequency parameter $\lambda = \omega a^2 \sqrt{\rho h / D}$ for a simply supported square plate with central cut-out (a/b=1.0, ʋ=0.3).

Cut-out ratio(c/a)	Mode no.	Proposed results
	1	19.421
	2	49.142
	3	49.142
	4	78.109
	1	19.098
	2	47.488
	3	47.488
	4	76.221
	1	19.259
	2	43.525
	3	43.525
	4	73.544
	1	20.707
	2	40.719
	3	40.719

	4	71.165
	1	23.223
	2	39.646
	3	39.646
	4	70.172

Here the effect of cut-out in an isotropic plate is given. From table 1 it is cleared that the cut-out ratio plays an important role in frequency. Here the cut-out ratio varies from 0.1 to 0.5. The first mode frequency is increases with the increase of cut-out ratio. But the second, third and fourth mode frequencies are decrease with increase the cut-out ratio.

B. Isotropic plate with various cut-out ratio (c/a) and thickness ratio (h/a).

The problem considers fundamental frequency of a square plate (a/b=1.0) of isotropic material (υ=0.3) having a central square cut-out (c*c) of various sizes for different thickness ratios (h/a). All edges are simply supported and the non-dimensional frequency is calculated as $\lambda = \omega a^2 \sqrt{\rho h / D}$.

Table-2:. Frequency parameter $\lambda = \omega a^2 \sqrt{\rho h / D}$ of isotropic square plate (a/b=1.0) with various cut-out ratio (c/a) and different thickness ratio (h/a).

Thickness ratio (h/a)		Cut-out ratio (c/a)				
		0.2	0.4	0.5	0.6	0.8
0.2		16.61	18.10	20.89	25.36	43.96
0.1		18.037	20.26	21.97	26.63	50.36
0.001		19.06	20.03	21.86	28.46	57.21

Here the result gives the changes of frequencies with the change the thickness ratio and cut-out ratio. For a fixed cut-out ratio the frequency is increased with decrease the thickness ratio. And when the thickness ratio is fixed the frequency increases with increasing the cut-out ratio.

C. Composite plate with different cut-out ratio (c/a):

A square laminate plate (a*a), (0/90) and (h/a=0.01) with different size of cut-out considered. The results have been compared for frequency parameter of $\lambda = \frac{\omega a^2}{h}\sqrt{\rho / E_2}$ show good agreement results as shown in table 3.

Material properties: E_1=25x10^{10} N/m^2, E_2=1x10^{10} N/m^2, G_{12}=G_{13}=0.5x10^{10}N/m^2, G_{23}=0.2x10^{10} N/m^2, υ_{12}=υ_{23}=0.25, ρ=1x10^{10} Kg/m^2

Table.3. Frequency parameter $\lambda = \frac{\omega a^2}{h}\sqrt{\rho / E_2}$ of simply supported, cross-ply (0/90), square laminate plate having rectangular cut-out.

Cutout size		Mode number					
		1	2	3	4	5	6
0.2a*0.2a		9.3	26.1	26.1	39.0	55.6	62.5
0.4a*0.4a		9.2	20.7	20.7	36.3	45.3	63.2
0.6a*0.6a		11.2	18.8	18.8	33.4	35.0	54.3
0.4a*0.2a		8.9	21.5	25.3	37.9	53.3	63.7
0.8a*0.4a		9.8	11.9	27.9	31.7	51.9	61.9
0.6a*0.2a		8.7	16.0	25.9	35.4	52.8	61.9

In this table 3 the effect of cut-out ratio in a composite cross-ply plate is given. The cut-out size for the first, second and third are square and the rests are rectangular. For the square cut-out, the frequencies are increase for the first mode for increase the cut-out size. But the second, third, fourth and fifth mode frequencies are decrease by increasing the cut-out ratio. For the rectangular cut-out the frequency is increase for the increase of the cut-out size.

D. Effect of material orthotropic property:

Three values of E1/E2 (i.e. the ratio of Young's modulus along and perpendicular to the fiber direction) are selected while other material properties are given below to study the effect of material orthotropic property on natural frequencies of the laminate for varying size of the cut-out.

Material properties: E_1/E_2=13 to 40, G_{12}=G_{13}=0.5x1010 N/m^2, G_{23}=0.33x10^{10} N/m^2, υ_{12}=0.35, ρ=1500kg/m^3. The non-dimensional frequency parameter is $\lambda = \frac{\omega a^2}{h}\sqrt{\rho / E_2}$.

Table.4. Frequency parameter $\lambda = \frac{\omega a^2}{h}\sqrt{\rho / E_2}$ of clamped edges, cross-ply (0/90/0/90), square laminate plate having different E_1/E_2 and varying c/a (c/b=0.4).

E_1/E_2	c/a	Mode(1)	Mode(2)	Mode(3)	Mode(4)
	0.1	21.1	29.9	39.4	51.1
	0.3	22.7	31.2	38.3	50.6
	0.5	27.3	35.2	38.1	50.4
	0.1	24.3	33.4	44.1	56.9
	0.3	26.1	35.1	37.1	53.4
	0.5	31.3	39.8	42.9	56.2
	0.1	30.2	39.4	51.8	66.1
	0.3	32.3	42.0	50.7	65.6
	0.5	38.3	47.6	51.0	65.5

From table 4 the effect of modulus ratio with cut-out ratio on frequency is studied. Here the modulus ratio changes from 13 to 40 and the cut-out is a rectangular cut-out with the size changes along x-axis from 0.1 to 0.5. The effect of cut-out size on frequency having varying modulus ratio is focused on the above discussions. Here it is clear that the frequencies are increase with increase of modulus ratio for a same cut-out ratio.

4. CONCLUSIONS

Because cut-outs are frequently utilised as access ports for mechanical and electrical systems, it is critical to anticipate the natural frequencies of laminate composite plates with cut-outs in the centre. In the presence of cut-outs, unwanted vibrations may induce unexpected failures owing to resonance. Because the plate has less mass, the natural frequencies increase when the cut-out size is increased, as shown in the above results. For isotropic and laminated plates, the non-dimensional fundamental frequency of vibration increases as the width to thickness ratio decreases. The fundamental frequency increases as the modulus ratio increases, reaching the maximum for the fourth mode.

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