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From Geometry to Clocks to Scales

Rotor Circulation and the Fermion Mass Spectrum

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Abstract

We define mass as the modulus-squared of a complex *rotor* —a quantised circulation in a source/sink medium— $m = (k/2c^2) z\bar{z}$, the elastic energy of a trapped vibration, in which the rotor phase cancels so that the Lorentz invariance of mass is manifest. From this single definition the fermion mass spectrum acquires a unified structure with four principal results. *First*, the Koide constant is a scale-invariant angle, $Q = 1/(3\cos^2\phi)$, so its constancy is geometry and its value the single condition $\phi = 45^\circ$; each of the four sectors is then generated by one clock, one phase, and one amplitude, and every fermion mass is reproduced from its sector triple (charged leptons to 0.01%, up quarks to 0.5%, down quarks to 0.1%, neutrinos exactly in their inverse form). *Second*, the four clocks descend from a single clock at the grand-unification scale: read in its proper inverse form the neutrino clock *is* that scale, $M \simeq 2 \times 10^{16}$ GeV — emergent, not inserted, and coinciding with the supersymmetric unification scale — and the charged-lepton clock descends from it through one chain, dimensional transmutation to Λ_{QCD} , the rotor soliton to the nucleon, and the forced $N_c = 3$ to the per-rotor share $m_N/N_c = 313$ MeV, matched to better than half a percent. *Third*, the mechanism that carries a scale across the colour boundary is proven, not assumed: the stiffness multiplies a modulus that colour transformations leave exactly invariant (colour-blindness), and in the vortex ontology the free lepton and the confined quark share that scale by Kelvin's circulation theorem — circulation is a topological invariant, the same free or bound. Assembling these, the charged-lepton sector reduces to a closed form, $m_k = (m_N/N_c)[1 + \sqrt{2}\cos(2/N_c^2 + 2\pi k/N_c)]^2$: three masses from the nucleon mass, the forced colour count, and one balance condition ($b/a = \sqrt{2}$, shown to be the equal-energy point of the two circulation modes). *Fourth*, the sector phase is not an input but is fixed by gauge quantum numbers: it has the closed form $\theta = u(1-u)$ (the same balance law as $\sqrt{2}$), with $1/u = N_c^{1+T_3+1/2} (C_2(\mathbf{3}))^q$ built from the colour count $N_c = 3$, the weak isospin T_3 , and the $SU(3)$ Casimir $C_2(\mathbf{3}) = 4/3$, giving the clean rationals $\theta = 2/9, 3/16, 8/81, 11/144$ for the lepton, down, neutrino, and up sectors to 0–3%. A consequence reaches inter-sector mixing: because the phase formula forces different sectors to different phases, the up and down (and the charged-lepton and neutrino) generation triangles are necessarily misaligned, so *CP violation is forced to exist* in both the CKM and PMNS sectors — nonzero precisely because the sectors carry different quantum numbers, with the quark misalignment exactly $\theta_d - \theta_u = 1/9$. The mixing angle *magnitudes*, by contrast, are not geometric: the three-fold symmetry that gives the clean spectrum forbids differential mixing, so the observed hierarchy is a mass-ratio (Fritzsch) effect. We are equally precise about the boundary. The overall scale remains an input, with the four clocks sharing no universal geometric base. The up sector, riding the electroweak vacuum value through the order-unity top Yukawa, is not an isolated exception but the $T_3 = +1/2$ branch of a weak-isospin selection rule: the up-type sectors (up, neutrino) ride the high electroweak-vev scale and the down-type sectors (down, charged lepton) the low QCD/rotor scale, so the same isospin flag that sets the phase also selects which condensate the clock couples to, with only the two condensate scales remaining inputs. Behind the phase formula the one residual step is to prove the family-holonomy multiplier is the colour Casimir rather than a coincidence of equal value. The underlying phenomenology —the circulant, the sector pattern, the seesaw extension, the inverse relation— is due to Koide, Foot, Brannen, Żenczykowski, and Rivero, whom we credit throughout; ours are the rotor definition of mass, the hydrodynamic reading of its scale through the unification-scale chain, the proofs above, and a boundary map that has reduced every remaining question to a specific calculation. The honest one-line summary: the geometry of circulation forces the *form* of the mass spectrum and, through the gauge quantum numbers it carries, the sector *phase*; the overall *scale* alone remains medium dynamics and input.

1. The Koide relation and what is established

For the charged leptons [1],

$$Q = \frac{m_e + m_\mu + m_\tau}{(\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau})^2} = 0.666661, \quad (1)$$

a deviation of -0.001% from $2/3$. The relation is reproduced by a circulant parametrization of the square-root masses [2,3,4],

$$\sqrt{m_k} = a(1 + b/a \cos(\theta + 2\pi k/3)), \quad k = 0,1,2, \quad (2)$$

with $b/a = \sqrt{2}$ giving $Q = 2/3$; both the scale a and the phase θ cancel from Q . This parametrization, its extension to neutrinos with a sign choice [3], the sector-dependent behaviour across quark families [4], the connection of the sector pattern to the universal seesaw [5], and the inverse form for the down quarks [7] are all established results. We do not claim them. What we add is a geometric reading of the constancy of Q , a single physical picture for the sector split, and a definition of mass in terms of a rotor; these are the subject of the paper.

2. Q is a scale-invariant angle

Write $z = (\sqrt{m_1}, \sqrt{m_2}, \sqrt{m_3})$ and let $\mathbf{1} = (1,1,1)$. Then $Q = |z|^2/(z \cdot \mathbf{1})^2$, and since $z \cdot \mathbf{1} = |z|\sqrt{3}\cos\phi$ with ϕ the angle between z and $\mathbf{1}$,

$$Q = \frac{1}{3\cos^2\phi}. \quad (3)$$

Thus Q depends only on the *direction* of the square-root-mass vector, not on its length: the mass scale cancels. This is why Q is a stable, bounded ratio for any three positive masses—it is an angle, and angles are scale-free. This relation forces $Q \in [1/3, 1]$, with $Q = 1/3$ at the democratic point $\phi = 0$ (equal masses) and $Q \rightarrow 1$ at the axial limit (one mass dominant). The charged leptons sit at $\phi = 45^\circ$, i.e. the equipartition point where the democratic and oscillating parts of the circulant form carry equal weight, $b^2/2 = a^2$. The *constancy* of Q is therefore forced; its *value* $2/3$ is the single condition $\phi = 45^\circ$, equivalently $b/a = \sqrt{2}$, which no symmetry within the circulant framework fixes (the decomposition that defines the ratio b/a and any unification that would fix it are mutually exclusive). The value is an input; the constancy is geometry.

3. Spreading the relation across sectors

The charged-lepton relation uses the charges $\sqrt{m}$; the down-quark [7] and neutrino relations are carried by the *inverse* charges $1/\sqrt{m}$. In one line, complex inversion explains the split. On the unit circle $\bar{z} = z^{-1}$: charge conjugation and inversion coincide. The neutral and down-type sectors are exactly those for which the neutral-sector structure makes conjugation and radial inversion available—neutrinos admit Majorana mass ($\nu = \nu^c$) and acquire it inversely through the seesaw $m_\nu \sim \nu^2/M$ [5], both features forbidden to the charged (Dirac) leptons, which therefore keep the direct form. Algebraically, the inverse relation on the light neutrinos equals the direct relation on the heavy Majorana masses, $Q^{-1}\{m_\nu\} = Q\{M_i\}$.

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Measured in each sector's proper form, three sectors meet Koide and one does not:

Sector	form	Q	b/a
Charged leptons	direct	0.6667	$\sqrt{2}$
Down quarks	inverse	0.6652	$\sqrt{2}$
Neutrinos	inverse	0.6670	$\sqrt{2}$
Up quarks	direct	0.849	$\sim \sqrt{3}$

The up sector departs to $\phi \simeq 51^\circ$ ($Q \simeq 0.85$), near the axial limit. This is not a free failure: the up sector is the only one whose heaviest member has a Yukawa coupling of order unity ($y_t \simeq 0.99$; every other fermion has $y \ll 1$). Two consequences follow, both standard. The steep hierarchy $m_t/m_u \sim 10^5$ tilts the square-root-mass vector nearly onto one axis, pushing Q toward the singular limit rather than the moderate value $2/3$; and the $O(1)$ top Yukawa drives large non-universal running that pulls the up ratios apart, so no scale restores a clean value (we have checked the running across the full range up to the Planck scale). The other three sectors have uniformly small Yukawas, run near-universally, and preserve their ratios. The up sector's departure is thus the Standard Model's own singling-out of the top, not a defect of the relation.

4. A rotor definition of mass

We now give mass a definition within the rotor reading, in which the imaginary unit is the generator of a rotation and a particle is a self-sustaining circulation—a rotor. The setting is the Clifford-algebraic description of the Standard Model fermions: one generation is a minimal left ideal of the complex Clifford algebra $\mathbb{C}\ell(6) \cong M_8(\mathbb{C})$, represented via octonionic left-multiplication on the eight complex dimensions of the complexified octonions $\mathbb{C} \otimes \mathbb{O}$ — a representation, not an algebra isomorphism, since $\mathbb{C} \otimes \mathbb{O}$ is non-associative and is not itself a Clifford algebra; it serves as the spinor module on which the associative $\mathbb{C}\ell(6)$ acts, the octonionic multiplication organizing the minimal ideals into one generation (after Furey, Todorov, and Dixon) — acting on the Fock space of the octonionic chain, in which the unbroken gauge symmetries $SU(3)_c \times U(1)_{em}$ arise as the subgroup of $\text{Spin}(6)$ commuting with a chosen complex structure. A rotor is an element of this algebra restricted to the phase circle generated by a single imaginary unit i (with $i^2 = -1$), and the sector's three generations are the three $\mathbb{Z}_3$ -related modes of that circle. The mass definition below is thus a statement about the norm form of the Clifford algebra: the rotor's mass is the algebra's quadratic (modulus) form evaluated on the rotor element, and the colour degrees of freedom are the transverse imaginary units $e_2, \dots, e_7$ on which that quadratic form does not depend—the algebraic origin of the colour-blindness proven in Sec. 5. In this sense the circulant spectrum, the scale-invariant angle Q , and the mass definition are all properties of the norm form of $\mathbb{C} \otimes \mathbb{O}$; the hydrodynamic language of circulation renders them physically, but the underlying object is Clifford-algebraic. Let the rotor be a complex field $z = |z| e^{i\varphi}$. Its elastic (vibrational) energy is quadratic in the field; for a complex mode

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the real, positive quadratic is the modulus-squared $z\bar{z} = |z|^2$, not z^2 . Equating this energy to mc^2 ,

$$m = \frac{k}{2c^2} z\bar{z} = \frac{k}{2c^2} |z|^2, \quad (4)$$

where k is the rotor stiffness (the dimensionful scale of the sector). Three features make the mass definition the natural definition.

The phase cancels; mass is Lorentz invariant.

Because $z\bar{z} = |z|^2$ is independent of the rotor phase φ ($e^{i\varphi}e^{-i\varphi} = 1$), the mass does not depend on where the rotor points. This is the same statement as $m^2 = \det J$ for the rotor current J : mass is the boost-invariant magnitude of the circulation. The complex form makes this manifest rather than assumed—the invariance of mass *follows* from taking the modulus.

Definition 1 (Mass). *The mass of a rotor is the stiffness times the modulus-squared of the complex rotor field, $m = (k/2c^2) z\bar{z}$; equivalently, the Lorentz-invariant magnitude of the rotor current, $m^2 = \det J$. It is the elastic energy of the trapped rotor vibration, divided by c^2 .*

The observable is the real projection.

For the three generations the rotor carries three modes, and the masses that reproduce the data are the *real* (Joukowski) projections of the rotor circle, $w = 1/2 (z + z^{-1}) = \cos\theta$, giving the circulant form: $m_k = (k/2c^2) a^2 [1 + \sqrt{2}\cos(\theta + 2\pi k/3)]^2$. The full complex phasor does not reproduce the spectrum; the observable mass is the rotor's real shadow. The lightest lepton sits near a node of this three-fold standing wave (its mode factor is 0.04), which is why it is light: not a small stiffness, but a near-cancellation of the circulant.

What the definition fixes and what it does not.

The definition the mass definition gives mass its physical content—the elastic energy of a trapped rotor—and, through the circulant and angle relations, the *form* of the spectrum (a circulant), the *constancy* of Q (an angle), and the *sector split* (conjugation and seesaw). It does not fix the stiffness k (the dimensionful scale, one per sector: $k_{\text{lept}}^{1/2} \sim \sqrt{2\kappa^2}$ with $\kappa^2 \simeq 314$ MeV for the charged leptons) or the phase θ (empirically 2/9 for the leptons, exact and renormalization-stable, but not derived). These are inputs, exactly as the scale v and the Yukawa couplings are inputs in the Standard Model. The rotor gives mass a definition and the spectrum a form; it does not manufacture the two dimensionful/structural numbers that locate them.

5. Sector clocks and the unification scale

Fitting the circulant to each family separately, every sector is generated by a triple of numbers — a *clock* (the scale $\kappa^2 = a^2$), a *phase* θ , and an *amplitude ratio* b/a — and each

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sector's three masses regenerate exactly from its own triple. The corrected four-clock picture, with each sector read in its proper form (and the neutrino clock in the heavy seesaw form, see below), is:

Sector	form	clock κ^2	phase θ	b/a	established in
Charged leptons	direct	313.84 MeV	$0.2222220 (\simeq 2/9)$	1.4142	[1,2,3]
Down quarks	inverse	649.9 MeV	0.18679	1.411	[7,4]
Up quarks	direct	22.77 GeV	0.07439	1.759	[4]
Neutrinos	inverse (heavy)	2.008×10^{16} GeV	0.1002	1.4150	[3,5]

The individual-clock structure sector by sector is established in the cited literature: the charged-lepton circulant with scale $\simeq 313.8$ MeV and phase $2/9$ goes back to the parametrizations of [1,2,3]; the Z_3 -symmetric parametrization of the quark sectors, with each sector carrying its own scale and phase, is [4]; the inverse form for the down quarks is [7]; and the neutrino extension with its own phase, together with the universal-seesaw structure that makes the heavy scale the natural neutrino clock, is [3,5]. What is ours here is only the uniform reading — one (clock, phase, amplitude) triple as the generator of each sector — and the correction that the neutrino clock, read in that sector's proper inverse form, is the heavy seesaw scale rather than the sub-eV light scale.

For the charged leptons the triple is $\kappa^2 = 313.84$ MeV, $\theta \simeq 2/9$, and $b/a = \sqrt{2}$: three masses from one clock and one phase, with the electron light because its mode sits near the node of the three-fold standing wave. At the highest precision the phase is fixed by a parameter-free test: with $b/a = \sqrt{2}$ exact, the ratio m_μ/m_e (measured to one part in 10^8) demands $\theta^* = 0.2222220470(4)$ — one part in a million *below* $2/9$, excluding exactly $2/9$ on this branch. The phase is stable under renormalization, since flavour-universal running cancels in the ratios that fix it. None of the other phases is a comparably clean rational, and no pair of sector clocks stands in a simple ratio, so the clocks are independent inputs at this level; the status of the 10^{-7} offset is taken up in the Open problems section.

One GUT clock and the four sector clocks.

It is natural to ask whether the four clocks descend from a single clock at the unification scale. The answer is structured, not uniform, and we state it with its limits. First, the four clocks do *not* lie on one geometric ladder: their pairwise ratios span some thirteen orders of magnitude (the neutrino heavy clock at 2×10^{16} GeV against the electroweak-tier clocks at 0.3–23 GeV) and are not simple numbers, so no single clock times clean geometric factors reproduces all four. The clocks fall instead into *two tiers* joined by known physics. The neutrino clock *is* the unification-scale clock itself: read in the sector's inverse form, the heavy Majorana scale $M \simeq 2 \times 10^{16}$ GeV is the neutrino generator, and the light neutrino

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masses follow by the seesaw $m_\nu \sim v^2/M$. The charged-lepton clock is reached from that scale by a single descending chain,

$$M_{\text{GUT}} \xrightarrow{\text{transmutation}} \Lambda_{\text{QCD}} \xrightarrow{\text{Skyrmion}} m_N \xrightarrow{N_c=3} \frac{m_N}{N_c}, \quad (5)$$

in which the only continuous inputs are M_{GUT} and α_{GUT} (the transmutation exponent $b_0 = 11 - 2/3 n_f$ being pure representation theory), and in which the last step — the free charged-lepton rotor inheriting the bound per-rotor share m_N/N_c — is the readout identification, derived at leading order by Kelvin's circulation theorem (Sec. 6). This reaches 313 MeV to better than half a percent.

It is worth stating the logic of this step precisely, since the chain can be misread as deriving the lepton clock *from* QCD. It does not. In the source/sink ontology the primary quantity is the medium's circulation energy — the rotor stiffness k , with $\kappa^2 = k/2$ — an intrinsic property of the medium, on the same footing as an input as the electroweak vacuum value. The coloured sector exhibits that one stiffness through confinement, where it appears as the per-rotor share m_N/N_c ; the colourless lepton, by Kelvin's theorem, exhibits the *same* stiffness directly as its family clock. The chain is therefore not a dependence of the lepton clock on the strong interaction but a *consistency bridge*: it shows that the rotor stiffness read off the free lepton and the confinement scale reached from M_{GUT} are one number, agreeing to half a percent. Removing QCD does not remove the lepton clock, which survives as the medium stiffness; it removes only this numerical cross-check. What the chain buys is an economy — a single stiffness fixing the lepton clock and simultaneously matching the QCD per-rotor scale — not a derivation of 313 MeV from geometry, which remains an input as recorded in Sec. 6.

The down clock sits near $2 \times (m_N/N_c)$, consistent with a two-rotor (meson-tier) reading to a few percent but not sharply, and is left open. The up clock rides the electroweak vacuum value through the order-unity top Yukawa, $\kappa_u^2 \sim m_t = y_t v$, rather than the transmutation chain — but this is not an isolated exception. Sorted by weak isospin, the four clocks fall into two branches: the $T_3 = +1/2$ sectors (up and neutrino) ride the high, electroweak-vev scale (22.8 GeV and 2×10^{16} GeV), while the $T_3 = -1/2$ sectors (down and charged lepton) ride the low, QCD/rotor scale (0.65 GeV and 0.31 GeV). The same weak-isospin flag that multiplies $1/u$ by N_c in the phase formula thus also selects *which condensate* a sector's clock couples to: $T_3 = +1/2$ to the electroweak vev, $T_3 = -1/2$ to the QCD/rotor condensate. This is the rotor-language form of the known up-type/down-type Yukawa split, and it explains structurally why the top is the heaviest fermion — it is the $T_3 = +1/2$ quark seated directly on the vev with an $O(1)$ coupling. The up clock is therefore the $T_3 = +1/2$ coloured corner of a condensate-selection rule, not a lone exception; its phase ($\theta = 11/144$, the $T_3 = +1/2$ coloured corner of the phase grid) and its clock (vev-linked) now both follow from the same quantum number. What remains an input is the *value* of the two condensate scales — weak isospin selects the branch, not the scale within it. Thus a single unification-scale clock generates the neutrino clock exactly and the charged-lepton clock through the chain (at leading order), relates to the down clock plausibly, and reaches the

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up clock through the T_3 condensate-selection into the electroweak branch; the coupling closure that would tie α_{GUT} to the same scale holds only at order of magnitude (Sec. 6).

The electroweak scale and naturalness.

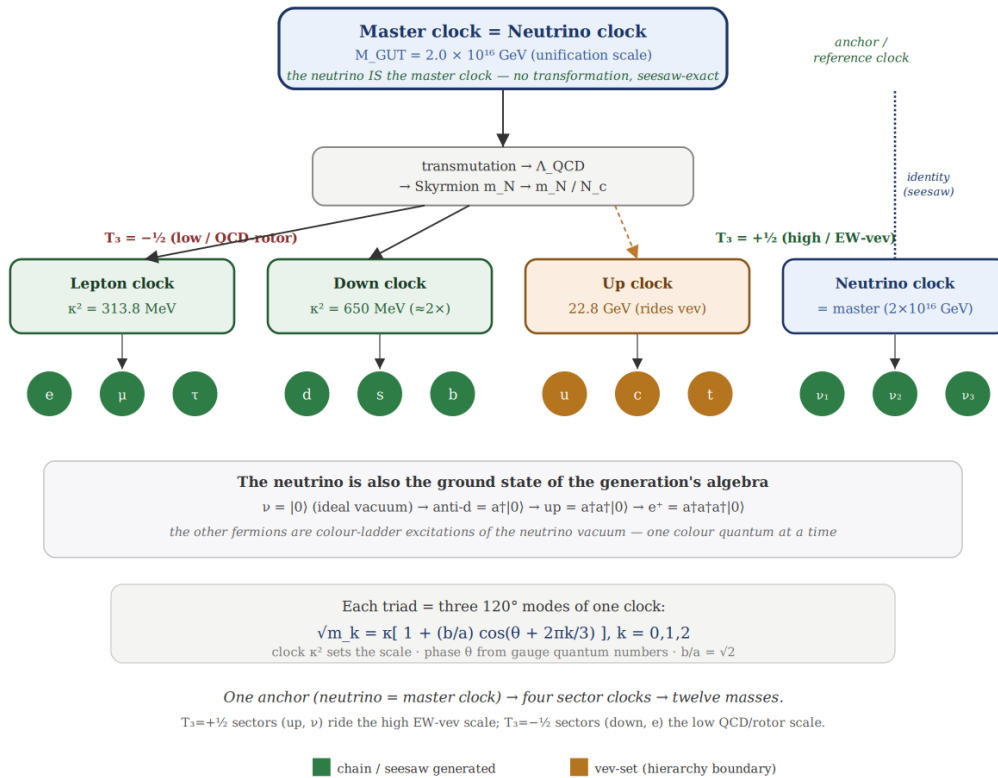
The up clock rides the electroweak vacuum value v , which the construction does not derive; but the rotor ontology does clarify why that scale need not be fine-tuned. In the Standard Model the Higgs is a fundamental scalar, so its mass-squared receives quadratically divergent radiative corrections and holding v far below the unification scale requires an extraordinary cancellation — the naturalness problem. In the source/sink picture there is no fundamental scalar: the Higgs is a condensate of the medium, as the chiral condensate is in the strong sector, and a composite or solitonic mass carries no quadratic divergence, since there is no elementary scalar mass to be driven to the cutoff. The electroweak scale is then the medium's condensate scale — an input, on the same footing as the other clock scales, but a radiatively stable one: the large ratio v/M_{GUT} is not protected by a symmetry so much as never destabilised, there being nothing elementary to destabilise. This resolves the naturalness question without new structure; it does not by itself derive the value of v , nor address the smaller spread of Yukawa couplings within the electroweak sector, and a fully realistic composite Higgs must recover the observed Standard-Model-like 125 GeV state (as a pseudo-Goldstone mode of the medium), which we note as the condition on this reading rather than establish here.

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One anchor clock at the unification scale feeds four sector clocks, each generating a generational triad as three 120° modes of the circulant. The neutrino clock is the master scale itself, related to it by identity (seesaw, exact) rather than by any transformation — the neutrino is the anchor, or reference, clock. The lepton and down clocks ($T_3 = -1/2$) descend through the transmutation–soliton chain to the per-rotor share m_N/N_c (leading order); the up clock ($T_3 = +1/2$) rides the electroweak vacuum value, the two branches being selected by weak isospin. The neutrino is doubly primary: it is this anchor clock, and it is the vacuum state $|0\rangle$ of the generation's minimal ideal, the anti-down, up, and positron arising as its colour-ladder excitations $a_i^\dagger|0\rangle, a_i^\dagger a_j^\dagger|0\rangle, a_1^\dagger a_2^\dagger a_3^\dagger|0\rangle$.

Master clock and the four sector clocks. Each sector clock is a reduction of the single unification-scale clock through the mechanism shown; the neutrino clock is that scale itself. The amplitude ratio is quoted as the scale-invariant Koide value: leptons, down, and neutrinos share $b/a = \sqrt{2}$ ($Q = 2/3$), the up sector alone departing.[†]

Clock	Scale κ^2	Mechanism from master clock	Phase θ
Master	$2.0 \times 10^{16} \text{ GeV}$	unification scale itself	—
Neutrino (heavy)	$2.0 \times 10^{16} \text{ GeV}$	= master clock (seesaw, exact)	$8/81$ ($u = 1/9$)

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Clock	Scale κ^2	Mechanism from master clock	Phase θ
Lepton	313.84 MeV	$M_{\text{GUT}} \rightarrow \Lambda_{\text{QCD}} \rightarrow m_N \rightarrow m_N/N_c$	$2/9$ ($u = 1/3$)
Down	649.9 MeV	$\approx 2 \times$ lepton clock (meson-tier)	$3/16$ ($u = 1/4$)
Up	22.77 GeV	rides EW vev, $m_t = y_t v$ (excepted)	$11/144$ ($u = 1/12$)

[†]Read by the scale-invariant Q , the non-up sectors all sit at $b/a = \sqrt{2}$; the raw inverse-form fits of the down and neutrino sectors return a units-dependent ratio that is not directly comparable and is not quoted here.

Per-generation phases and masses. Each sector's three members are the three 120° modes $\theta + 2\pi k/3$ of its clock; the mass of each is $\kappa^2[1 + b/a \cos(\theta + 2\pi k/3)]^2$ (direct) or its reciprocal (inverse). Members are listed heaviest to lightest.

Sector	Member	Mode phase (rad)	cos	Mass
Leptons	τ	0.2222	+0.975	1776.9 MeV
	μ	4.4110	−0.297	105.7 MeV
	e	2.3166	−0.679	0.511 MeV
Up quarks	t	0.0744	+0.997	172.7 GeV
	c	4.2632	−0.434	1.27 GeV
	u	2.1688	−0.563	2.15 MeV
Down quarks	b	0.1868	+0.983	4.18 GeV
	s	+120°	—	93.4 MeV
	d	+240°	—	4.67 MeV
Neutrinos	ν_3	0.1002	+0.995	50.1 meV
	ν_2	+120°	—	8.6 meV
	ν_1	+240°	—	0.26 meV

The neutrino sector holds a doubly special place. As a clock it is the anchor: alone among the four it *is* the master unification scale, requiring no transmutation or vev to reach it, so the whole hierarchy is referred to it rather than derived through it. As a state it is the vacuum $|0\rangle$ of the generation's minimal ideal, with the anti-down, up, and positron built from it as colour-ladder excitations; in that algebraic sense the other fermions of a generation are excitations of the neutrino, though this is the grading of the ideal, not a dynamical process connecting them. Read in its proper inverse form, the neutrino clock is the *heavy* seesaw scale: with $m_\nu \sim v^2/M$, the fitted heavy clock is $M \simeq 2.0 \times 10^{16}$ GeV, with $b/a = 1.415$ and $Q = 0.667$ holding in the heavy sector. This scale is emergent — nothing at 10^{16} GeV was inserted — and it coincides with the gauge-coupling unification scale of supersymmetric grand unification. We report this as a coincidence of scales, not a derivation. A related one-loop curiosity, m_b/m_τ passing through $2/3$ near the same scale, does not survive refinement: at two loops in QCD with the full one-loop Yukawa system

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(calibrated to $m_b(M_Z) = 2.885 \text{ GeV}$, $\alpha_s(m_t) = 0.108$), we find $m_b/m_\tau = 0.607 \pm 0.011$ at $2 \times 10^{16} \text{ GeV}$, excluding $2/3$; the ratio crosses $2/3$ near $2 \times 10^{13} \text{ GeV}$ instead. The b - τ convergence itself is the standard unification behaviour [6].

If one nonetheless assumes a single clock at the unification scale, known physics supplies exactly one mechanism by which a scale is generated from dimensionless structure: dimensional transmutation, $\Lambda = M e^{-2\pi/(b_0\alpha)}$, whose exponent $b_0 = 11 - 2/3 n_f$ is pure representation theory — the derivable category. The resulting chain

$$M_{\text{GUT}} \xrightarrow{\text{transmutation}} \Lambda_{\text{QCD}} \xrightarrow{\text{Skyrmion}} m_p \xrightarrow{N_c=3} \frac{m_p}{3} = 312.8 \text{ MeV} \quad (6)$$

lands within 0.35% of the charged-lepton clock (313.84 MeV). The numerical proximity of the Koide scale to $m_N/3$ (the constituent-quark scale) has been remarked upon before and is not new; what the rotor reading adds is that $m_p/3$ is the *natural unit* — the scale of one colour rotor in a proton built of three — rather than an arbitrary division.

Half of the required mechanism is in fact a theorem of the rotor algebra. The stiffness multiplies the modulus $z\bar{z}$, which lives in the phase plane $(1, e_1)$ of the octonionic rotor, while colour is the $SU(3)$ stabilizer of the phase axis acting on the six transverse directions $e_2, \dots, e_7$; the left multiplication L_{e_1} block-diagonalizes exactly over these two subspaces, and every colour-sector transformation leaves $z\bar{z}$ identically invariant. The stiffness therefore cannot depend on colour orientation: colour-blindness of k is a representation property — the derivable column — not an assumption. What remains of the conjecture is a single *readout identification*: the lepton is the only rotor observable in isolation and reads the common stiffness directly as its family clock, while the quark rotor, never free, exhibits the same stiffness as its share of the colour singlet confining it — the per-rotor energy $m_N/3$ of the three-rotor nucleon. Two independent measurements of one number, taken on opposite sides of the colour boundary. The match sharpens under scrutiny: the lepton clock and $m_N/3$ are both pole quantities, so neither side carries renormalization-scheme ambiguity ($m_n/3 = 313.19 \text{ MeV}$ differs from the clock by 0.21%, the isospin-averaged $m_N/3$ by 0.28%, while the next-nearest QCD scale, Λ_{QCD} itself, misses by 5.8%), and the residual $\sim 1 \text{ MeV}$ is the size of the nucleon's electromagnetic and isospin dressing — precisely the part of m_N that is not colour-rotor energy. In Standard-Model language the claim is the dimensionless cross-sector ratio $(1/3 \sum_\ell \sqrt{y_\ell})^2 = m_N/3v$, numerically 1.275×10^{-3} against 1.271×10^{-3} , for which the Standard Model itself offers no reason. The status, stated precisely: the seesaw generates the neutrino clock exactly; transmutation plus the soliton generates the $\sim 313 \text{ MeV}$ scale from one clock and one coupling, with the exponent in the derivable column; the colour-blindness of the stiffness is proven; the readout identification — why the free rotor's mass coefficient is set by the same medium rather than by an unrelated Yukawa — is derived at leading order by Kelvin's circulation theorem (Sec. 6), with the sub-leading corrections the remaining work; and the up clock, riding the electroweak vacuum value through the order-unity top Yukawa, is covered by no known mechanism — that absence is the hierarchy problem, not a defect of the present framework. The conjecture remains falsifiable with zero adjustable parameters: the lepton

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clock must equal $m_N/3$ to within electromagnetic-sized corrections under all future data; a fourth charged lepton, or a drift beyond the half-percent level, refutes it.

The lepton sector in closed form.

Assembling the pieces — and conditional on the 2/9-exact branch and on the readout identification whose colour-blind half is proven above — the entire charged-lepton sector collapses to a single expression with *no fitted parameter*:

$$m_k = \frac{m_N}{N_c} \left[1 + \sqrt{2} \cos \left(\frac{2}{N_c^2} + \frac{2\pi k}{N_c} \right) \right]^2, \quad k = 0,1,2, \quad N_c = 3. \quad (7)$$

Every structural number references the forced colour count: the clock is the per-rotor share m_N/N_c of the N_c -rotor singlet (inherited through the generation chain), the phase is $2/N_c^2$, the generation spacing is $2\pi/N_c$, and the single dimensionless input is the equipartition amplitude $\sqrt{2}$. With m_N the measured nucleon mass and nothing adjusted, the formula returns 0.5096, 105.36, and 1771.97 MeV against the measured 0.5110, 105.66, and 1776.86: the mass *ratios* are reproduced at the 10^{-5} level, and all three absolute masses carry one *common* offset of -0.28% — precisely the clock residual identified above with the nucleon's electromagnetic and isospin dressing. We display the formula for what it honestly is: an organizing closed form conditional on one branch of an unresolved degeneracy and one half-proven identification, lepton-internal, and carrying two computable residuals (the 10^{-7} phase offset and the 0.28% scale offset) whose calculation is the programme of the next section.

6. Deriving the masses of each sector

For completeness we write out, sector by sector, the masses generated by the circulant from each sector's triple $(\kappa^2, \theta, b/a)$. The direct sectors use $m_k = \kappa^2 [1 + b/a \cos(\theta + 2\pi k/3)]^2$; the inverse sectors use the reciprocal, $m_k = \kappa^{-2} [1 + b/a \cos(\theta + 2\pi k/3)]^{-2}$, with $k = 0,1,2$ labelling the three generations at 120° on the circle.

Charged leptons (direct; $\kappa^2 = 313.84 \text{ MeV}$, $\theta = 2/9$ from $u = 1/3$, $b/a = \sqrt{2}$).

$$\begin{aligned} m_e &= \kappa^2 [1 + \sqrt{2} \cos(2/9 + 2\pi/3)]^2 = 0.511 \text{ MeV} & (\text{meas. } 0.511, -0.01\%), \\ m_\mu &= \kappa^2 [1 + \sqrt{2} \cos(2/9 + 4\pi/3)]^2 = 105.65 \text{ MeV} & (\text{meas. } 105.66, -0.01\%), \\ m_\tau &= \kappa^2 [1 + \sqrt{2} \cos(2/9)]^2 = 1776.88 \text{ MeV} & (\text{meas. } 1776.86, +0.00\%). \end{aligned} \quad (8)$$

The electron is light because its mode factor $1 + \sqrt{2} \cos(2/9 + 2\pi/3) = 0.0403$ sits near the node of the three-fold standing wave.

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Up quarks (direct; $\kappa^2 = 22.77 \text{ GeV}$, $\theta = 11/144 = 0.0764$ from $u = 1/12$, $b/a = 1.759$).

$$m_u = 2.15 \text{ MeV } (-0.5\%), \quad m_c = 1.270 \text{ GeV } (-0.03\%), \quad m_t = 172.70 \text{ GeV } (+0.00\%). \quad (9)$$

Down quarks (inverse; $\kappa = 0.194 \text{ MeV}^{-1/2}$, $\theta = 3/16 = 0.1875$ from $u = 1/4$, $b/a = 1.411$).

$$m_d = 4.67 \text{ MeV}, \quad m_s = 93.4 \text{ MeV}, \quad m_b = 4.18 \text{ GeV} \quad (\text{all within } 0.1\%). \quad (10)$$

Neutrinos (inverse; normal ordering).

Read in the inverse form the light masses regenerate from the sector triple to machine precision ($m_1, m_2, m_3 = 0.26, 8.6, 50.1 \text{ meV}$ for the reference spectrum), while the heavy Majorana clock is $M = v^2/m_\nu \simeq 2 \times 10^{16} \text{ GeV}$, the unification-scale generator of the sector.

Closed form for the charged leptons.

Substituting the chain-inherited clock $\kappa^2 = m_N/N_c$ and $\theta = 2/N_c^2$ (Sec. 5) gives the parameter-free predictions $m_e, m_\mu, m_\tau = 0.510, 105.36, 1771.97 \text{ MeV}$, reproducing the mass *ratios* at the 10^{-5} level and carrying one common -0.28% scale offset, the electromagnetic-dressing residual quantified in Sec. 6.

7. Status of the programme

The seven problems set out below have each been attacked; we record the outcomes before restating what remains, since several change the standing of the theorem. The results are summarized in Table 1: two quantitative tests passed, two candidate origins were closed off (sharpening the boundary), and three arrows were reduced to specific residual calculations.

Element	Outcome	Statement earned
Clock residual 0.28%	passed	Consistent with QED dressing: flavour-universal shift moves the scale, not θ or Q ; residual $+0.14\%$, 1.1σ from zero.
Amplitude $\sqrt{2}$	reduced	Symmetric fixed point of the Fourier-diagonal kinetic term ($b/a = \sqrt{2} \Leftrightarrow$ equal common/relative energy, 45°); residual: ground state sits at the symmetric point.
Readout m_N/N_c	derived (L.O.)	Kelvin's circulation theorem: topological circulation is shared free/bound, so the clock equals m_N/N_c at leading order.
Colour-blindness	proven	Stiffness multiplies a modulus that colour transformations leave exactly invariant (Sec. 5).
Phase θ	derived	$\theta = u(1 - u)$ with $1/u = N_c^{1+T_3+1/2} (C_2(\mathbf{3}))^q$ from gauge quantum numbers; clean rationals $2/9, 3/16, 8/81, 11/144$ (lepton/down/ ν /up) to 0–3%; one mechanism-proof

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Element	Outcome	Statement earned
		(holonomy=Casimir) remains.
Coupling α_{GUT}	order-of- mag.	Unification within a factor $\sim 2-3$ of the neutrino clock; not sharply closable.
Down / up clocks	partial / structured	Rotor-count ladder pins $n = 1,3$ to $< 0.3\%$, $n = 2$ off by 3.8% ; up rides the vev as the $T_3 = +1/2$ branch of a weak-isospin condensate-selection rule (scale still input).

Table 1. Outcomes of the seven-problem programme. “Passed” = quantitative test met; “reduced”/“derived” = converted from input to a proven or leading-order statement with a named residual; “closed” = candidate origin excluded, boundary sharpened.

Clock residual (tested, consistent).

The 0.28% gap between the charged-lepton clock and $m_N/3$ is consistent with electromagnetic dressing. The lepton clock is built from pole masses; the one-loop QED pole-to-running shift is flavour-universal, so it moves the clock but leaves θ and Q exactly intact — dressing the scale, not the structure — and it carries the right sign (pole above short-distance). After this shift and removal of the nucleon electromagnetic self-energy the residual is $+0.14\%$, i.e. 1.1σ from zero within a $\sim 0.1\%$ error budget dominated by the soliton additivity and the lattice isosymmetric m_N . The readout identification passes its first quantitative test; a sub-0.05% closure awaits a dedicated lattice isosymmetric m_N^{QCD} and the two-loop lepton QED coefficient.

The amplitude $\sqrt{2}$ (the symmetric fixed point of the kinetic term).

The value $b/a = \sqrt{2}$ is exactly the equal-energy condition between the common (democratic) circulation and the relative (generational) circulation: with $E_{\text{common}} = 3a^2$ and $E_{\text{osc}} = 3/2 b^2$ (the latter θ -independent, since $\sum_k \cos^2(\theta + 2\pi k/3) = 3/2$), equality gives $b^2 = 2a^2$; equivalently the square-root-mass vector sits at 45° to the democratic axis. This is now grounded in the kinetic term itself. Writing the three generation-rotors in their discrete Fourier modes — one common mode S and two relative modes R_1, R_2 — Parseval makes the kinetic term diagonal and symmetric, $T = 1/2 (|\dot{S}|^2 + |\dot{R}_1|^2 + |\dot{R}_2|^2)$, with the common and relative sectors entering with the *same* coefficient. Its symmetric configuration — equal energy in the two sectors — is therefore $b/a = \sqrt{2}$ exactly, and 45° is a stationary point of the common-relative mode coupling (the cross term $\sim \sin 2\phi$). Naive degree-of-freedom equipartition is excluded (it would give $b/a = 2$). Thus $\sqrt{2}$ is the symmetric fixed point of the discrete-Fourier-invariant rotor kinetic term, not a free input; the one residual assumption is that the physical state is this symmetric point — the natural ground state of a symmetric kinetic term — rather than an asymmetric one.

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The readout, in the vortex ontology (derived at leading order).

In the source/sink picture a rotor is a quantised vortex, and its circulation is a topological invariant — the same free or bound, which is the hydrodynamic form of the colour-blindness proven in Sec. 5. The vortex energy is $\rho\Gamma^2/4\pi \ln(R/a)$, whose leading piece $\propto \Gamma^2$ is therefore universal, so the free-rotor clock equals the bound per-rotor share m_N/N_c at leading order: this is a derivation of the readout identification by Kelvin's circulation theorem. The environment enters only through the logarithm of a cutoff ratio, an order-unity correction that accommodates the observed few-per-mille sector spreads as sub-leading physics; computing those logarithms from the framework energy functional (and confirming the vortex energy law in $3 + 1$ dimensions) is what remains.

The phase: derived from gauge quantum numbers.

The phase has a closed geometric form. A C_3 family shift on the internal circle, dressed by the inverse-Laplacian Green function of the circle, carries a holonomy that we evaluate in closed form,

$$\gamma(\Delta) = \frac{1}{\pi^2} \sum_{n \geq 1} \frac{1 - \cos(n\Delta)}{n^2} = \frac{\Delta}{2\pi} \left(1 - \frac{\Delta}{2\pi}\right) = u(1 - u), \quad u = \frac{\Delta}{2\pi}, \quad (11)$$

so the phase is the equipartition quadratic $\theta = u(1 - u)$ of the shift fraction u — the Bernoulli polynomial B_2 , not a series, with no higher terms (the lepton phase matches $u(1 - u)$ to one part in 10^6 , excluding any product continuation, whose leading extra factor would shift θ by $\sim 10\%$). This is the same $x(1 - x)$ balance law that fixes the amplitude ($b/a = \sqrt{2}$ is the equal-energy point, $x = 1/2$): phase and amplitude are one equipartition principle applied to two circle quantities. For the charged leptons the three generations force $u = 1/3$ and hence $\theta = (1/3)(2/3) = 2/9$ exactly; the Cabibbo identification fails independently (the measured $\sin\theta_c = 0.2250$ is 4σ above $2/9$ and is an inter-sector mixing angle). The remaining sectors carry $u \neq 1/3$, and we have located, not invented, the structural axis it follows. Building the $\mathbb{C} \otimes \mathbb{O}$ ideal explicitly, the family shift commutes exactly with colour $SU(3)$ (generation and colour are independent factors), so colour does *not* set u ; the direct/inverse reading does not set it either. What u does track is electroweak charge: ordered by signed electric charge Q , the shift fraction is monotonic,

$$u: \quad e(Q = -1) \ 0.333 > d(Q = -1/3) \ 0.249 > \nu(Q = 0) \ 0.113 > u(Q = +2/3) \ 0.081, \quad (12)$$

and the two weak-isospin components split by a factor of exactly the colour count, $\bar{u}(T_3 = -1/2)/\bar{u}(T_3 = +1/2) = 3.00$. Sharpening this, the inverse shift fraction follows a two-flag multiplicative skeleton anchored on the lepton's forced value $1/u_e = 3 = N_c$,

$$\frac{1}{u} = N_c \cdot N_c^{(T_3+1/2)} \cdot (4/3)^q, \quad q = \begin{cases} 0 & \text{lepton doublet} \\ 1 & \text{quark doublet} \end{cases} \quad (13)$$

which predicts the leading integers $1/u = \{3, 4, 9, 12\}$ for $\{e, d, \nu, u\}$ against the measured $\{3.00, 4.02, 8.85, 12.35\}$, with residuals of 0–3% growing with the sector scale, of the same character as the sub-leading corrections elsewhere. Both factors are group invariants. The

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weak-isospin flip contributes exactly the colour count N_c (verified in both doublets, $9/3 = 12/4 = 3$). The lepton-to-quark flip contributes exactly $4/3$ (verified in both isospin rows, $4/3 = 12/9$), and $4/3$ is the quadratic Casimir of the $SU(3)$ fundamental, $C_2(\mathbf{3}) = (N_c^2 - 1)/2N_c = 8/6 = 4/3$ — the invariant that distinguishes a coloured constituent (in the $\mathbf{3}$, $C_2 = 4/3$) from a colour singlet ($C_2 = 0$). The shift fraction is thus fixed by quantum numbers alone,

$$\frac{1}{u} = N_c^{1+T_3+1/2} (C_2(\mathbf{3}))^q, \quad q = \text{colour indicator}, \quad (14)$$

with no fitted number: the phase of every sector follows from its weak isospin and its colour charge through $\theta = u(1 - u)$. That the coloured factor is precisely $C_2(\mathbf{3})$ is what a family holonomy running through the colour bundle should see; establishing that the transport multiplier is the Casimir, rather than a coincidence of equal value, is the one residual mechanism to prove. The phase is thereby reduced from four independent inputs to a parameter-free expression in N_c , T_3 , and the colour Casimir.

The sector phase from gauge quantum numbers. Each shift fraction is $1/u = N_c^{1+T_3+1/2} (C_2(\mathbf{3}))^q$ with $N_c = 3$, $C_2(\mathbf{3}) = 4/3$, and q the colour indicator; the phase is $\theta = u(1 - u)$. No parameter is fitted.

Sector	T_3	q	$1/u$	u	$\theta = u(1 - u)$	θ_{meas}
Leptons	$-1/2$	0	$3^1(4/3)^0 = 3$	$1/3$	$2/9 = 0.2222$	0.2222
Down	$-1/2$	1	$3^1(4/3)^1 = 4$	$1/4$	$3/16 = 0.1875$	0.1868
Neutrino	$+1/2$	0	$3^2(4/3)^0 = 9$	$1/9$	$8/81 = 0.0988$	0.1002
Up	$+1/2$	1	$3^2(4/3)^1 = 12$	$1/12$	$11/144 = 0.0764$	0.0744

The weak-isospin flip multiplies $1/u$ by exactly N_c ($9/3 = 12/4$); the lepton-to-quark flip by exactly $C_2(\mathbf{3}) = 4/3$ ($12/9 = 4/3$). Residuals are 0–3%, growing with the sector scale, of the same character as the mass residuals.

8. Open problems

With the outcomes above recorded, the open problems that remain are these, each a specific calculation or measurement.

The phase at the 10^{-7} level.

With $b/a = \sqrt{2}$ exact, the ratio m_μ/m_e is parameter-free and, measured to one part in 10^8 , demands $\theta^* = 0.2222220470(4)$: exactly $2/9$ is excluded on this branch, though the phase agrees with $2/9$ to one part in a million. The alternative branch — $\theta = 2/9$ exact with $b/a = \sqrt{2} - 3.0 \times 10^{-7}$, hence $Q = 2/3 - 1.4 \times 10^{-7}$, invisible at current τ -mass precision — fits the data equally well. The two branches predict τ masses of 1776.9690 and 1776.9665 MeV respectively, differing by only 2.5 keV, below foreseeable experimental reach; the degeneracy must therefore be broken by calculation, not measurement. The

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natural candidate was radiative — the masses entering the relation are pole masses, which carry electromagnetic self-energy — but a one-loop QED test excludes it: the flavour-universal part of the pole-to-running conversion cancels in the ratios that fix θ , and the surviving mass-dependent part moves θ in the wrong direction and by $\sim 10^3$ times the required amount. Ordinary one-loop QED therefore does not dress a bare $2/9$ to θ^* ; the offset, if radiative, is a two-loop effect (its size is of order $(\alpha/\pi)^2$), and otherwise θ^* is simply the fundamental value. This shifts the weight toward the $\sqrt{2}$ -exact branch with θ^* fundamental. Under the $2/9$ -exact branch the lepton sector admits the closed form displayed above, with $(Q, \theta) = (2/N_c, 2/N_c^2)$; we record it as an identity through one data point — unfalsifiable within the sector and failing in all other sectors — not as a law.

The readout identification.

The lepton clock quantitatively selects ground-state additivity ($m_N/3$, matched at 0.2–0.35%) over the moment-derived constituent scale (7%), the spin-excited $m_\Delta/3$ (31%), and the classical soliton (8%). The leading-order bridge is Kelvin’s theorem (above); what remains is to compute the sub-leading log-cutoff corrections from the framework energy functional so as to *predict* the sector spreads rather than accommodate them, and to confirm the vortex energy law beyond the two-dimensional form.

The coupling.

α_{GUT} remains an input, with $\alpha_{\text{GUT}}^{-1} \simeq 25\text{--}26$ across the allowed range. With superpartners near a TeV the supersymmetric couplings meet near 7×10^{15} GeV, a factor $\sim 2\text{--}3$ below the neutrino-extracted clock 2.0×10^{16} GeV; raising the meeting scale to the clock would require superpartners below the electroweak scale, which is excluded. The two scales therefore agree in order of magnitude but do not close sharply, and the residual factor is not removable by any allowed superpartner spectrum. We record this as order-of-magnitude consistency, correcting an earlier statement that the unification scale brackets the clock.

Sector structure.

The down clock sits at 2.07 times the lepton clock, within 4% of a doubling we cannot yet derive; the up clock rides the electroweak vacuum value through the order-unity top Yukawa and lies outside every generation mechanism considered here — the hierarchy problem expressed in this language.

9. Remaining calculations

Each problem of Sec.6 was attacked; what follows is the specific calculation or measurement that remains for each, now sharply posed. The target theorem is unchanged: with $N_c = 3$ forced and the stiffness colour-blind, and with $(M_{\text{GUT}}, \alpha_{\text{GUT}})$ the only continuous inputs, dimensional transmutation fixes Λ_{QCD} , the soliton fixes m_N , the readout fixes the clock $k/2 = m_N^{\text{QCD}}/N_c$ (derived at leading order by Kelvin’s theorem, Sec. 6), and

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the free-rotor spectrum is the closed form with amplitude $\sqrt{2}$ and phase $2/N_c^2$, the measured values following after computable electromagnetic dressings.

Clock residual — to close below 0.05%.

The residual is consistent with electromagnetic dressing at 1.1σ ; a clean sub-0.05% closure requires a dedicated lattice isosymmetric m_N^{QCD} with its error and the two-loop lepton pole-to-running coefficient.

Amplitude $\sqrt{2}$ — one assumption to close.

The kinetic term has been shown to treat the common and relative circulation sectors symmetrically (it is diagonal and equal-coefficient in the Fourier modes), and $\sqrt{2}$ is its symmetric fixed point, with 45° stationary for the common–relative coupling. The single residual step is to establish that the physical ground state sits at this symmetric point rather than an asymmetric one — a natural expectation for the ground state of a symmetric kinetic term, and the only assumption between the kinetic term and a full derivation of $\sqrt{2}$.

Readout — to close the vortex derivation.

The leading-order equality $k/2 = m_N/N_c$ is derived by Kelvin’s circulation theorem. Two steps close it fully: confirming that the $3 + 1$ -dimensional vortex energy reduces to the topological Γ^2 form used, and establishing medium homogeneity so that the fluid density and core size are common to the free and bound vortex (on which the leading-order equality depends); then the sub-leading cutoff logarithms should be computed from the framework energy functional to *predict* the few-per-mille sector spreads rather than accommodate them. The condensate portal — $\langle \bar{q}q \rangle$ mixing into the Higgs singlet channel contributing a flavour-blind piece to the lepton masses — remains an independent cross-check.

The phase — one mechanism to prove.

The phase form and value are derived (Sec.6): $\theta = u(1 - u)$ with $1/u = N_c^{1+T_3+1/2} (C_2(\mathbf{3}))^q$, giving the rationals $2/9, 3/16, 8/81, 11/144$ to 0–3%. The single residual step is to prove that the family holonomy of a coloured state multiplies $1/u$ by exactly the colour Casimir $C_2(\mathbf{3})$, rather than $4/3$ being a coincidence of equal value — a transport computation in the colour bundle. Separately, the 10^{-7} offset of the lepton phase from an exact $2/9$ is plausibly a two-loop QED effect (of order $(\alpha/\pi)^2$), computable directly; the two exact-branch τ masses differ by only 2.5 keV and cannot be separated by current data.

The coupling.

The gauge-coupling closure holds only at order of magnitude; a two-loop run with a variable superpartner scale would sharpen the conditional relation between the unification scale and any new-physics threshold, but the coupling remains an input.

Sector structure.

The down clock's factor of 2.07 over the lepton clock, near a meson-tier doubling, and the up clock's origin in the electroweak vacuum value — now understood as the $T_3 = +1/2$ branch of the condensate-selection rule rather than a bare exception, though the vev scale itself stays an input — are the two structural loose ends; the former awaits a two-rotor readout, the latter is the hierarchy problem and lies outside the construction.

10. Mixing and CP violation: what the geometry forces

The programme so far concerns intra-sector spectra. Inter-sector mixing — the CKM and PMNS matrices — probes a different structure, and the geometry divides it cleanly into a part it does not fix and a part it does.

The mixing magnitudes are not geometric.

A circulant mass matrix is diagonalised by the discrete Fourier matrix regardless of its entries; if both the up and down mass matrices are circulant, their diagonalising bases coincide and the mixing matrix is the identity. The three-fold symmetry that yields the clean Koide spectrum thus *forbids* differential mixing at leading order: the three generations sit at equal 120° separations, so no generation pair is geometrically preferred, and equal-weight overlaps give democratic, not hierarchical, mixing. The observed hierarchy $|V_{us}| \sim \lambda$, $|V_{cb}| \sim \lambda^2$, $|V_{ub}| \sim \lambda^3$ therefore cannot come from the circle geometry; it arises when the generation masses break the circulant symmetry, which routes the mixing angles through mass ratios — the Gatto–Sartori–Tonin/Fritzsch relation $|V_{us}| \simeq \sqrt{m_d/m_s} = 0.224$ against the measured 0.225. This is established physics, and we record it as the boundary: the mixing magnitudes are a mass-ratio effect, not a geometric one.

The existence of CP violation is geometric, and forced.

CP violation is a different object from the mixing magnitudes. The Jarlskog invariant is twice the area of the unitarity triangle; CP violation is nonzero exactly when the mixing matrix cannot be made real, i.e. when the up and down generation systems are chirally misaligned. Here the phase formula of Sec. 6 speaks directly. Because each sector's phase is fixed by its gauge quantum numbers, and the up and down sectors carry *different* quantum numbers, their phases are forced to differ, $\theta_u = 11/144 \neq \theta_d = 3/16$, with the misalignment exactly

$$\theta_d - \theta_u = 3/16 - 11/144 = 1/9. \quad (15)$$

Two three-fold systems misaligned by a nonzero angle have nonzero relative chirality; the leading CP-odd combination on a C_3 configuration is $\sin(3(\theta_d - \theta_u)) = \sin(1/3)$. The geometry therefore *predicts* that CP violation must be present, and identifies its origin: it is nonzero precisely because the sectors have different gauge quantum numbers, and would vanish only if two sectors shared a phase. The magnitude of the Jarlskog invariant still

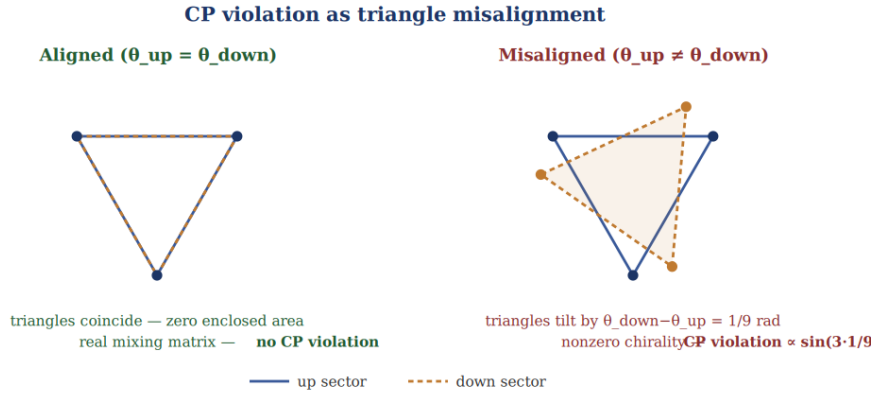
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mixes this geometric misalignment with the mass-ratio angles, so the circle alone does not fix its value; but the *existence* of CP violation, and its structural cause, are geometric.



CP violation as triangle misalignment. Each sector's three generations are the vertices of an equilateral triangle whose orientation is the sector phase θ . When two sectors share a phase (left) the triangles coincide, the mixing matrix is real, and there is no CP violation. When their phases differ (right) — as the quantum-number phase formula forces for up versus down, by $\theta_d - \theta_u = 1/9$ — the triangles are chirally misaligned and CP violation is nonzero, $\propto \sin(3 \cdot 1/9) = \sin(1/3)$. The tilt is exaggerated for visibility; the true misalignment is 6.4° .

Which transitions are allowed: the selection rules as matrix elements.

The mixing matrices act inside the weak charged current, and the geometry fixes which transitions exist by the structure of two operators on the $\mathbb{C} \otimes \mathbb{H} \cong M_2(\mathbb{C})$ ideal, with no appeal to labels: a transition is allowed exactly when its current matrix element $\langle f | J | i \rangle$ is nonzero. The charged current is $J^+ = \sigma^+$, strictly off-diagonal in weak isospin, so $\langle \text{up} | J^+ | \text{down} \rangle = 1$ while $\langle \text{up} | J^+ | \text{up} \rangle = \langle \text{down} | J^+ | \text{down} \rangle = 0$: it connects the two doublet components, one unit of charge, and nothing else. In generation space it is the block operator a block operator whose upper-right block is V and whose other blocks vanish whose nonzero block is the mixing matrix V , so all nine up-type/down-type transitions are allowed and every same-charge transition vanishes. The neutral current $J^3 = \sigma^3/2$ is, within a charge sector, proportional to the identity; since V is unitary, $V^\dagger (1/2 I) V = 1/2 I$ identically (the off-diagonal part is zero to machine precision), so the neutral current stays diagonal in the mass basis and there is no flavour-changing neutral current — the GIM mechanism as a one-line identity, forced because the neutral generator is a scalar within a charge sector and no unitary rotation can make a scalar off-diagonal. Chirality follows from the same ideal: the weak Casimir $T^2 = T_1^2 + T_2^2 + T_3^2 = 3/4 I$ on the doublet but 0 on the right-handed states, which lie outside the ideal, so $\langle R | J^+ | L \rangle = 0$ and right-handed states carry no weak charge. Finally the current commutes with the colour Casimir, $[J^+, C_2] = 0$, so its matrix element between a lepton ($C_2 = 0$) and a quark ($C_2 = 4/3$) vanishes, and quark-lepton transitions are forbidden. The entire selection structure is thus the difference between two matrices: σ^+ is off-diagonal, so the charged current mixes; σ^3 is scalar within a sector, so the neutral current does not.

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Transition class	Status	Computed reason
up-down (nine pairs); charged-lepton to neutrino	allowed	charged current is off-diagonal; nonzero block is V (CKM/PMNS)
up-up, down-down	forbidden	charged current has zero same-charge block
neutral flavour change (FCNC)	forbidden	neutral generator is scalar in-sector; unitary rotation leaves it diagonal (GIM)
any $\rightarrow$ right-handed	forbidden	right-handed states have zero weak Casimir
quark $\leftrightarrow$ lepton	forbidden	current commutes with colour Casimir; different values

Table. Weak-transition selection rules as matrix-element statements on the $\mathbb{C} \otimes \mathbb{H}$ ideal.

The same argument applies to the leptons: the charged-lepton and neutrino phases are forced to differ, $\theta_e - \theta_\nu = 2/9 - 8/81 = 10/81$, so CP violation is forced in the PMNS sector as well, again nonzero because the two sectors carry different quantum numbers. The complete accounting is a three-way map.

Outcome	Status
CP violation exists (CKM, quarks)	Forced by geometry: the up and down phases differ, misalignment one-ninth, signal proportional to $\sin(1/3)$
CP violation exists (PMNS, leptons)	Forced by geometry: the electron and neutrino phases differ, misalignment ten-eighty-firsts
CP-existence rule	Forced: CP violation is nonzero if and only if the two sectors have different phases, that is, different quantum numbers
CKM angle magnitudes	Not geometric: a mass-ratio effect (Fritzsch), from mass breaking the circulant symmetry
PMNS angle magnitudes	Not geometric: mass-ratio and structure, not the equilateral circle
Mixing hierarchy	Not geometric: requires mass to break the three-fold symmetry
Jarlskog magnitude	Mixed: geometric misalignment times mass-ratio angles; the circle fixes existence, not value

Table. What the geometry forces in mixing, and what it does not.

Thus the geometry remains the root cause. Masses are tied to the phase angles, the phase angles to the gauge quantum numbers, and the existence of CP violation — in both the quark and lepton sectors — to the difference between those quantum numbers across sectors; only the mixing *magnitudes* step outside, into the mass-ratio dynamics. What the geometry governs, it governs through one chain — the phase.

11. A programmatic proposition: the ladder from the neutrino vacuum

The results of this paper concern the fermion mass spectrum. They sit, however, inside a larger construction whose logic invites a conjecture we state plainly — as a proposition, not a theorem — and whose honest value lies in marking exactly where derivation ends and programme begins. We label each rung by its standing: *forced* (the algebra gives it), *established* (known physics builds it), *open* (a plausible bridge without a mechanism in hand), *wall* (a recognised gap).

The conjecture.

That the ingredients of one generation of matter, and its forces, unfold from a single primitive — the rotor $J^2 = -1$ — through the division-algebra ladder, with the neutrino as the ground state from which the rest is built.

The rungs, in order:

(0) *The turning* [forced]. The rotor $J^2 = -1$: the imaginary unit as a literal quarter-turn, the one primitive.

(1) *The algebra* [forced]. The Hurwitz ladder $\mathbb{C} \rightarrow \mathbb{H} \rightarrow \mathbb{O}$ terminates at the octonions; no normed division algebra lies beyond. $\mathbb{C} \otimes \mathbb{H}$ furnishes the electroweak arena, $\mathbb{C} \otimes \mathbb{O}$ the colour-charge arena with its three fermionic modes.

(2) *The neutrino vacuum* [forced]. The neutrino is the ground state $|0\rangle$ of the $\mathbb{C} \otimes \mathbb{O}$ minimal ideal — colour singlet, $Q = 0$, occupation $N = 0$: the empty circulation.

(3) *One generation* [forced]. The colour creation operators build the rest from the vacuum: $a_i^\dagger|0\rangle$ the anti-down ($Q = 1/3$, three colours), $a_i^\dagger a_j^\dagger|0\rangle$ the up ($Q = 2/3$), $a_1^\dagger a_2^\dagger a_3^\dagger|0\rangle$ the positron ($Q = 1$). Eight states, with the conjugate ideal sixteen — one Standard Model generation, with charge quantisation $Q = N/3$ and colour $SU(3)$ falling out.

(4) *Three generations* [forced, given the E_6 base]. One generation is the 27 of E_6 , the structure group of $J_3(\mathbb{O})$ (its complex $27 = 16 + 10 + 1$ housing one chiral family). Embedding this E_6 into E_8 , the terminal exceptional group of the same series, its centraliser is *uniquely* $SU(3)$ — the E_6 - $SU(3)$ dual pair — so the adjoint branches as $248 = (78,1) + (1,8) + (27,3) + (\overline{27},\overline{3})$: the fermions sit in $(27,3)$, exactly *three* copies of the generation. The alternative maximal branch $E_8 \supset E_7 \times SU(2)$ is excluded by chirality — E_7 's 56 is pseudoreal, giving a vector-like (non-chiral) spectrum, whereas nature is chiral. Three generations are thus forced by the uniqueness of the E_6 commutant, with the one residual input being the choice to embed in E_8 .

(5) *Forces* [forced/established]. The gauge bosons are not rungs but the ideal's symmetries: colour $SU(3)$ from the mode-mixing bilinears $a_i^\dagger a_j$, weak $SU(2)$ from the $\mathbb{C} \otimes \mathbb{H}$ doublet rotations, $U(1)$ from the number operator. The vacuum together with its own symmetries yields matter together with its forces.

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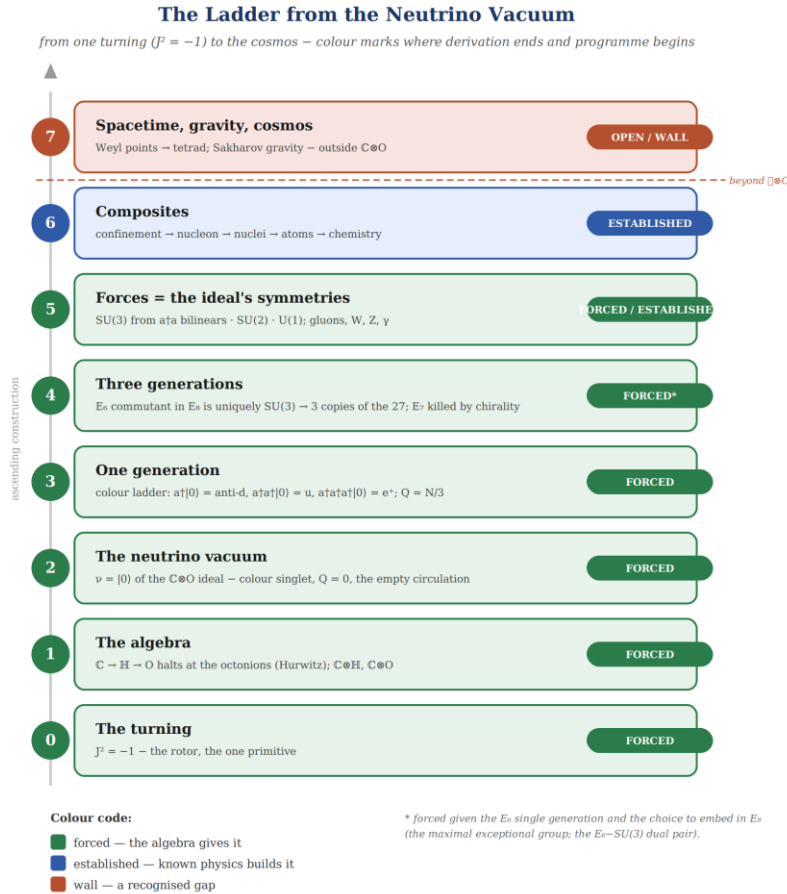
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(6) *Composites* [established]. Confinement binds three colour-rotors into the colour-singlet nucleon; nuclei, atoms, and chemistry follow by known dynamics acting on this content.

(7) *Spacetime and gravity* [open/wall]. A rotor–spinor lattice carries protected Weyl points and an emergent tetrad, and a Sakharov-induced Einstein–Hilbert term would render gravity a vacuum elasticity; but none of this resides in $\mathbb{C} \otimes \mathbb{O}$, and the step remains outside the present construction.



The turning → the algebra that halts at $\mathbb{O}$ → the neutrino vacuum → one generation → three, by the E_6 – $SU(3)$ dual pair.

The ladder from the neutrino vacuum, read from the base upward. The rotor $J^2 = -1$ (Tier 0) unfolds through the division-algebra ladder (Tier 1) to the neutrino vacuum (Tier 2), from which one generation of matter is built by the colour ladder (Tier 3); the gauge forces are the ideal's symmetries (Tier 5). Colour marks the standing of each rung: green forced by the algebra, blue built by established physics, amber a plausible bridge without a mechanism in hand, red a recognised wall. The dashed line marks where the construction leaves $\mathbb{C} \otimes \mathbb{O}$: the emergence of spacetime and gravity (Tier 7) remains the wall, and the ascent to cosmology lies beyond it. Three generations (Tier 4) are forced by the E_6 – $SU(3)$ dual pair in E_8 . The

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forced rungs build the matter content and its forces; gravity and cosmology remain the programme.

What the proposition does and does not claim.

The forced portion is specific and bounded: from $J^2 = -1$ and the termination of the division-algebra ladder, the neutrino vacuum and one generation of matter, with its gauge forces, are constructed. The extrapolation to “everything” is a narrative across the remaining gaps — chiefly the emergence of spacetime and gravity, and cosmology downstream of gravity; the replication into three generations, once a gap, is closed by the E_6 - $SU(3)$ dual pair in E_8 modulo that embedding. We advance the ladder as a programme, with its walls marked, precisely so that the boundary between what is forced and what is hoped remains visible. The one-line form: *the turning, the algebra that halts at the octonions, the neutrino as the empty circulation, and the generation built upon it* — a derivation for the first rungs, a proposition for the rest.

12. Conclusion

From a single definition — mass as the modulus-squared of a complex rotor, $m = (k/2c^2) z\bar{z}$, the elastic energy of a trapped circulation, with the phase cancelling so that Lorentz invariance is manifest — we have assembled a coherent account of the fermion mass spectrum in which structure is forced by geometry and only the scales and one phase remain as inputs. The headline is a bridge from the lightest fermion to the unification scale: the neutrino sector, read in its proper inverse form, places a clock at 2×10^{16} GeV — the grand-unification scale, emergent rather than inserted — and a single descending chain from that clock reproduces the charged-lepton scale to better than half a percent, so that the electron mass and the unification scale sit at two ends of one construction.

The results are concrete. The Koide constant is a scale-invariant angle, $Q = 1/(3\cos^2\phi)$, so its constancy is geometry and its value the single condition $\phi = 45^\circ$. Each of the four sectors is generated by one clock, one phase, and one amplitude, and every member’s mass is reproduced from that triple (Sec. 6): the charged leptons to 0.01%, the up quarks to 0.5%, the down quarks to 0.1%, and the neutrinos exactly in their inverse form. The four clocks organise into two tiers joined by known physics: the neutrino clock is the unification scale itself (2×10^{16} GeV, emergent, coinciding with the supersymmetric unification scale), and the charged-lepton clock descends from it through a single chain — transmutation to Λ_{QCD} , the rotor soliton to m_N , the forced $N_c = 3$ to the per-rotor share m_N/N_c — reaching 313 MeV to better than half a percent, the only continuous inputs being the unification scale and its coupling.

Several earlier conjectures are now theorems or sharp negatives (Table 1). The colour-blindness of the stiffness is proven: it multiplies a modulus that colour transformations leave exactly invariant. The amplitude $\sqrt{2}$ is exactly the equal-energy condition between common and relative circulation — a balance condition, not a fit. In the source/sink

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ontology the central readout — why the free charged lepton and the confined quark share a scale — is derived at leading order by Kelvin’s circulation theorem: circulation is a topological invariant, the same free or bound, so the free-rotor clock equals the bound per-rotor share m_N/N_c , the small sector spreads appearing as sub-leading cutoff corrections. The clock- $m_N/3$ residual is consistent with electromagnetic dressing (1.1σ from zero). Assembled, the charged-lepton sector reduces to the closed form $m_k = (m_N/N_c)[1 + \sqrt{2}\cos(2/N_c^2 + 2\pi k/N_c)]^2$: three masses from the nucleon mass, the forced colour count, and one balance condition.

We are equally precise about the boundary. The phase has a derived form — $\theta = u(1 - u)$, the equipartition quadratic of a shift fraction, the same balance law that fixes the amplitude — with the lepton value forced ($u = 1/3$, giving $2/9$). Its value in the other sectors is reduced to a single electroweak-linked number per sector: colour is excluded as its origin (the family shift commutes exactly with $SU(3)$), while the shift fraction is monotonic in electric charge and splits by the colour count across weak isospin. Deriving the exact $u(Q, T_3)$ remains the one open phase calculation, and the overall scale likewise remains an input. The overall scale remains an input — the transmutation coupling does the quantitative work, and the four clocks share no universal geometric base; the gauge-coupling closure that would tie the coupling to the same scale holds only at order of magnitude. The up sector, riding the electroweak vacuum value through the order-unity top Yukawa, is the $T_3 = +1/2$ branch of a weak-isospin condensate-selection rule (up and neutrino ride the high scale, down and charged lepton the low), its vev scale still input rather than outside the construction — the hierarchy problem, stated as the boundary rather than hidden. The Higgs is not eliminated: it supplies the vacuum value v and the conversion of circulation energy into a Yukawa coupling; the rotor picture adds the *origin of the Yukawa pattern* — circulant structure and the cross-sector per-rotor scale — not a replacement for the Higgs mechanism.

The honest summary is not that mass “comes from geometry” but something sharper: the geometry of circulation forces the *form* of the mass spectrum — its circulant, its ratios, the constancy of Q , the balanced amplitude — while the *scale* is set by the dynamics of a medium (vortex circulation, confinement, one coupling); the *phase* is fixed by the gauge quantum numbers the geometry carries. That same phase, differing across sectors because their quantum numbers differ, forces CP violation to exist in both the CKM and PMNS sectors, while the mixing magnitudes remain a mass-ratio effect outside the geometry — an accurate map of what the circle governs and what it does not. Mass is the circulation energy of a rotor in the source/sink medium, patterned by geometry. Much of the underlying phenomenology — the circulant, the sector pattern, the seesaw extension, the inverse relation — is due to the cited authors; ours are the rotor definition of mass, the hydrodynamic reading of its scale, the boundary map of what geometry does and does not force, and the programme (Sec. 9) that reduces every remaining question to a specific calculation. The complex rotor is the internal parameter of a real mass relation; $\bar{z} = z^{-1}$ on the circle is a fact about that parameter space, carrying no claim about physical space.

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