

Author : Randeep Chaudhary

Independent Research

B604 Ramkrishan CGHS, Plot 12, Dwarka, Sector 23, Delhi 110077

Contact: +91 9971396196, Email:[randeep.chaudhary@gmail.com; randeep@pouchnation.com; randeep.chaudhary@magiclenz.co.in]

Iota (i) as a Rotation in the 2D and 3D Complex Planes

A Unified Thesis on Iota, Giving Rise to the Standard Model of Particles

Randeep Chaudhary

Principal Systems Architect — Independent Research

randeep.chaudhary@gmail.com

Every claim in this thesis is tagged FORCED, INPUT, OPEN, or NEGATIVE. Every figure and every number was produced by direct computation and is quoted together with its residual.

Abstract

Notation. C , H and O denote the complex numbers, the quaternions and the octonions, and $J_3(O)$ the exceptional Jordan algebra. Plain capitals are used in place of blackboard-bold so that the text renders identically on every system.

This thesis advances a single claim and follows it to its limit: i (iota) is a projection. It is the residue left in our arithmetic wherever a rotation belonging to an inner complex world has been cast down into the three real dimensions we inhabit. That i (iota) appears in every fundamental equation of physics is therefore not a curiosity of notation. It is the signature of a world we observe only in outline.

The casting occurs in two stages. A 3 Dimensional inner Complex World, in which spheres turn, casts its shadow onto 2 Dimensional inner Complex Planes, in which circles turn; those planes cast their shadow in turn onto the 3 Dimensional Real World our instruments measure. Both stages are exact maps rather than metaphors: $S^5 \rightarrow CP^2$ at the first, and the Hopf map $\psi \rightarrow \psi^\dagger \sigma \psi$, carrying $S^3 \rightarrow S^2$, at the second. Each stage removes one complete circle of phase, and that circle is not lost bookkeeping. It is the electric charge. We see the shadows and take them for the objects; i (iota) is the record of everything the projection could not carry down.

This inner structure is a medium, and it is the medium the aether hunters were right to expect and wrong about in one respect only — its address. It possesses no rest frame and no extension, because it does not lie within space; it is attached to every point of space. An interferometer measures distances within space, so the experiment of 1887 could not have reached it, and its failure was never evidence against it. The intuition was sound. The location was not.

i (iota) is not a single value but a collection. Inside a fixed algebra the solutions of $u^2 = -1$ form S^0 in C , S^2 in H , and S^6 in O , verified here on twenty thousand samples in each case. Selecting one point of that six-sphere is the same act as breaking G_2 to $SU(3)$, since $14 - 8 = 6$ exactly, and colour is therefore the residue of a choice made inside i (iota) itself. On the real line the same object appears as an orbit: the square-root map returns real numbers for a negative radicand, and its trajectories are cyclic, quasiperiodic, or unbounded. i (iota) is all of those points at once.

From this foundation the ladder $C \rightarrow H \rightarrow O \rightarrow J_3(O)$ is forced by Hurwitz's theorem, admitting four algebras and no fifth, and it delivers the entire architecture of the Standard Model: charge quantisation together with its integrality, the fractions 0 , $\frac{1}{3}$, $\frac{2}{3}$ and 1 , spin- $\frac{1}{2}$, weak $SU(2)$, chirality, colour $SU(3)$, $N_c = 3$, baryon number, $Q = T_3 + Y/2$, and one generation's chiral 27 of E_6 . Every gauge group, every charge, every spin and every representation follows from the single requirement that the inner multiplication preserve norm without leakage.

The boundary of the result is stated with the same precision as the result itself. The architecture is forced; the measured numbers are not, and this thesis identifies the exact line between them. i (iota) is necessary for quantum mechanics to exist at all — real quantum theory is not merely inelegant but experimentally excluded — and it is the origin of every quantum number carried by the topology of the phase: charge, angular momentum, spin- $\frac{1}{2}$, circulation and baryon number. Knowing precisely where the forcing ends is not a limitation of this work. It is the work.

Prologue: the picture

The sections that follow make a picture precise. This one describes the picture.

A sphere turning in a room casts a moving disc upon the wall. An observer who has only ever seen the wall will construct a complete and predictive science of discs, and that science will work. It will also contain one persistent oddity: a quantity that appears in every equation, that squares to a negative, that has no length upon the wall and cannot be pointed at. That quantity is not a flaw in the arithmetic of discs. It is the sphere, seen edge-on from a world with no room to hold it.

This thesis argues that we occupy precisely that position, and it argues it with named maps rather than with metaphor. There is a 3 Dimensional inner Complex World in which spheres turn. It casts its shadow upon 2 Dimensional inner Complex Planes in which circles turn. Those planes cast their shadow upon the 3 Dimensional Real World in which our instruments sit. At each casting a complete circle of phase is removed, and i (iota) is what that removal leaves behind.

We observe the shadows and never notice the circles that cast them; and those circles are themselves the shadows of spheres turning in a richer inner world. i (iota) is the mark of both cuts, carried down into our arithmetic because the geometry had nowhere else to leave it.

Three consequences follow. Each is stated plainly here and established with its exact boundary later.

- **The medium is real, and no interferometer could ever have found it.** The aether hunters were right that physics is written upon something. They searched for it within space. It is attached to space — an inner world at every point, possessing no rest frame, no extent and no location, which is exactly why the arm of an interferometer cannot cross it. Section 10 renders this as a table rather than an assertion.
- **i (iota) is a collection, not a value.** In the octonions the square roots of minus one form a six-sphere. Upon the real line the same object appears as an orbit — cyclic, quasiperiodic or unbounded — and no single real number is the answer. i (iota) is all of those points simultaneously, and Section 9 shows that the freedom to choose among them is the origin of colour.
- **The particle is not a point.** Writing the wavefunction in modulus-and-phase form converts quantum mechanics into a fluid exactly, both residuals verified to be identically zero. No point hides within the cloud; the equations contain no such variable. The cloud is the electron, and what measurement returns is its shadow.

1. The five theses and their standing

Thesis	Standing	Exact form
i (iota) has structure, and isotopes	FORCED	$u^2 = -1$ yields S^0, S^2, S^6 ; and the anchor family, whose members differ in angle, period and energy

Thesis	Standing	Exact form
Periods 3, 4 and 6	FORCED	the crystallographic restriction at rank 2; five-fold order opens at rank 4
i (iota) originates the standing wave	FORCED	a wave is stationary precisely when its rotation closes, in every setting where waves stand
i (iota) is the reason for quantization	FORCED	necessary for quantum mechanics to exist at all, and the origin of every quantum number carried by the topology of the phase
i (iota) gives rise to the Standard Model	ARCHITECTURE FORCED	every group, every charge, every spin and every representation is forced

The five theses are developed in that order. Each is accompanied by the computation that establishes it, and the ledger of Section 17 records the standing of every claim in the document together with the ground on which it rests.

2. What i (iota) is

i (iota) is defined by the single property $i(iota)^2 = -1$. Everything in this thesis follows from reading that property geometrically rather than arithmetically.

Read as a number, i (iota) is a quantity whose square is negative and which therefore has no place upon the real line. Read as an operator, i (iota) is the generator of a quarter-turn within an oriented plane: the real operator J satisfying $J^2 = -I$, whose flow carries a vector around a circle at constant magnitude. Two applications of a quarter-turn produce a half-turn, and a half-turn is multiplication by minus one. The defining equation is thus not a paradox to be tolerated but a description of what the operator does.

The second reading is not an interpretation placed upon the first. A one-dimensional linear law $u' = ku$ admits only growth, decay or rest, and no real constant produces bounded oscillation. The minimal real structure capable of oscillating while conserving magnitude is a plane carrying a J with $J^2 = -I$, and by Frobenius's theorem the real plane equipped with such a J is uniquely the complex numbers. The algebraic definition and the geometric one therefore coincide, and they coincide by necessity rather than by convention.

i (iota) is a quarter-turn. Every appearance of i (iota) in physics marks a place where a rotation was carried out in a plane our instruments cannot see, and the equation retained the turning even though the world it describes retained only the shadow.

Tag: FORCED. The equivalence of the two readings is a theorem of real linear algebra together with Frobenius.

3. The function: real numbers from a negative root

Let x be a real anchor and a a real parameter. The generator of the whole construction is

$$y_{n+1} = x + a / (x + y_n), \quad y_0 = x,$$

which is the classical square-root continued fraction $\sqrt{x^2+a} = x + a/(2x + a/(2x + \dots))$ rewritten as a recurrence. When $x^2 + a > 0$ the convergents approach the real root in the ordinary way. The construction becomes interesting in the regime $x^2 + a < 0$, where the target is imaginary and yet every term of the sequence is real.

Writing $x^2 + a = -d^2$, the update is the Möbius action of the matrix $M = [x, x^2+a; 1, x] = x \cdot I + d \cdot J$, where $J = [0, -d; 1/d, 0]$ satisfies $J^2 = -I$ exactly. M therefore behaves as the complex number $x + i \cdot d$. Its fixed points are $\pm i \cdot d$, its eigenvalues are $x \pm i \cdot d$, and the map is consequently elliptic — conjugate to a rigid rotation. The orbit admits a closed form, matched to machine precision against direct iteration:

$$y_n = \sqrt{|x^2+a|} \cdot \cot((n+1)\alpha), \quad \alpha = \arctan(\sqrt{|x^2+a|}/x).$$

i (iota) is not eliminated by working upon the real line. It sits inside M as the real operator J , and it survives the entire computation, re-encoded as the rotation angle α .

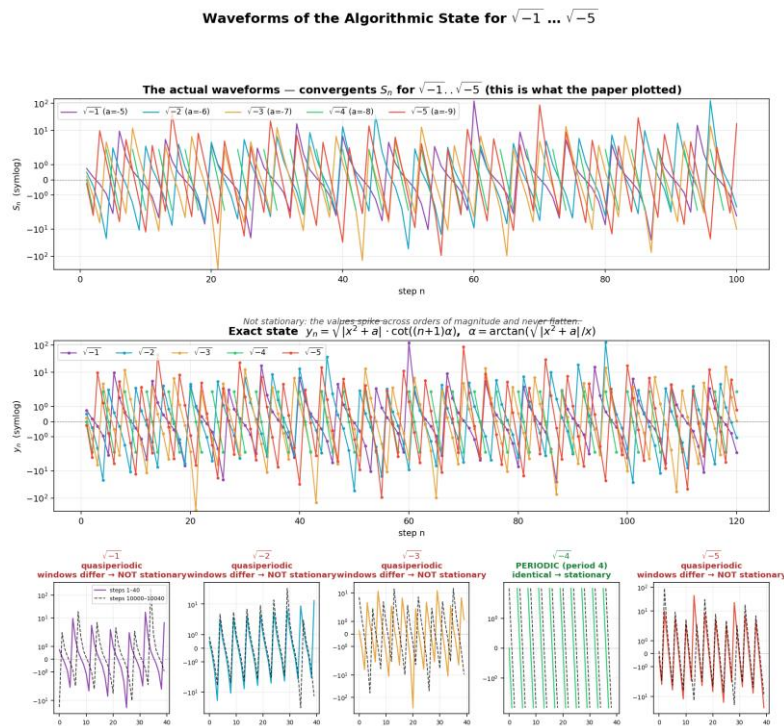


Figure 1. Convergents (top) spike across orders of magnitude; the exact state (middle); a stationarity test (bottom) in which only the periodic $\sqrt{-4}$ is identical in early and late windows.

The middle panel is the decisive one. What the top panel presents as violent instability is a single cotangent sampled at equal angular steps, and the apparent chaos is an artefact of viewing a rotation from the side. The bottom panel supplies the discipline: comparing an early window against a late one, only the genuinely periodic case is unchanged. Everything else drifts, however convincing it appears over a short run.

$$\text{Iota as a real-valued orbit: } y_n = \sqrt{|x^2 + a|} \cot((n+1)\alpha), \quad x^2 + a < 0$$

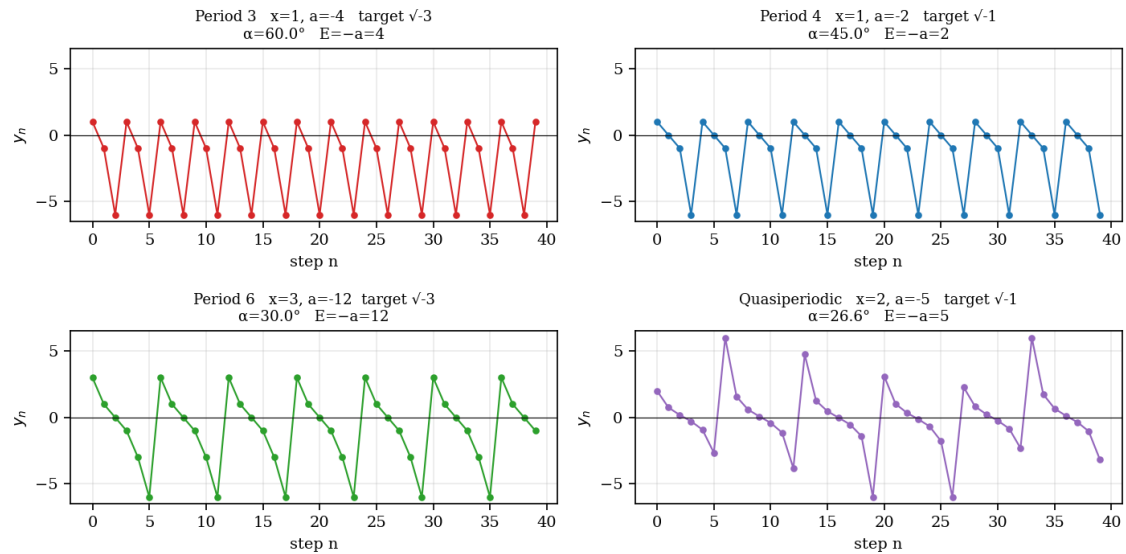


Figure 2. The same function in four regimes. Closing orbits at periods 3, 4 and 6 — the only periods integer data admits — together with one quasiperiodic orbit that never repeats. All four are real sequences targeting an imaginary root.

The three closing families are exact and infinite. Setting $a = -2x^2$ produces period 4 with block $\{x, 0, -x, \infty\}$; $a = -4x^2$ produces period 3 with block $\{x, -x, \infty\}$; and $a = -4x^2/3$ with 3 dividing x produces period 6 with block $\{x, x/3, 0, -x/3, -x, \infty\}$.

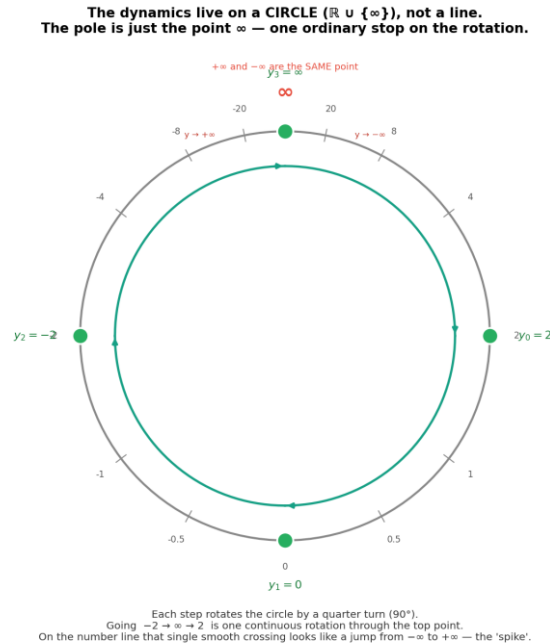


Figure 3. The extended real line as a circle; the pole is the point ∞ , one ordinary stop upon the rotation.

The pole is not a numerical failure. The dynamics live upon the extended real line, which is a circle, and once in every cycle the rotation passes through the point at infinity as smoothly as through any other point. Flattening that circle onto a line is what converts one ordinary crossing into an apparent infinite spike. The spikes of the top panel of Figure 1 are the one-dimensional shadow of a smooth circular

passage — which is the thesis of this document in miniature, at the simplest level on which it can be demonstrated.

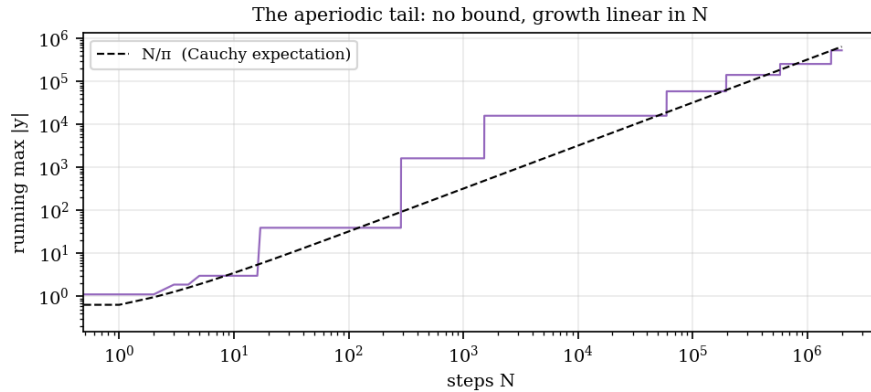


Figure 4. The aperiodic case admits no bound. The running maximum grows linearly in N , tracking the expectation N/π , and is still climbing after two million steps.

4. The isotopes of i (iota)

The same target root is reached from many anchors, and those anchors are not equivalent to one another. Each carries its own rotation angle, its own period and its own conserved energy:

$$\alpha = \arctan(\sqrt{|x^2+a|}/x), \quad E = -a = x^2 \sec^2 \alpha.$$

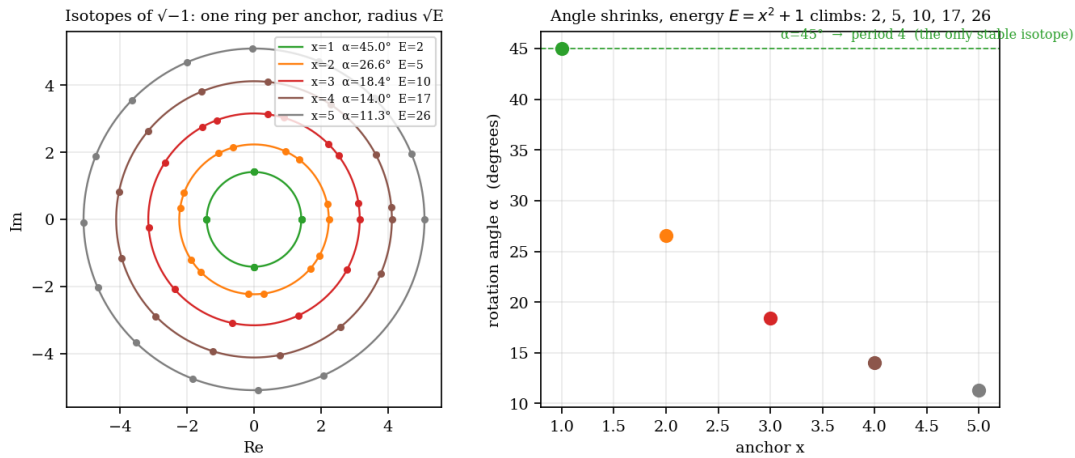


Figure 5. Isotopes of $\sqrt{-1}$. Each anchor rides its own conserved-energy ring of radius $\sqrt{E}$, advancing at its own angle. Exactly one closes: $x = 1$ at $\alpha = 45^\circ$, the period-4 orbit $\{1, 0, -1, \infty\}$. The remainder are quasiperiodic and progressively heavier, at $E = 2, 5, 10, 17$ and 26 .

The analogy holds where it matters. The same target is reached with differing internal structure; one stable form stands among an infinite unstable series; and the distinguishing labels — angle, period and energy — play the role of a mass number. Even the criterion of stability has the right mechanism, since the stable isotope is the one whose rotation closes.

The three elements behave differently, and the law behind the difference is exact. $\sqrt{-1}$ possesses one stable isotope, at $x = 1$. $\sqrt{-3}$ possesses two, at period 3 from $x = 1$ and at period 6 from $x = 3$. $\sqrt{-2}$ possesses none at any integer anchor, since closure would require $x = \sqrt{2}$ — an element every one of whose isotopes is unstable. In general $\sqrt{-K}$ admits a closing integer anchor if and only if K or $K/3$ is a perfect square.

Tag: **FORCED**. The classification is complete and was verified against direct iteration for every family.

5. Three, four and six — and what lies beyond them

The orbit closes precisely when α/π is rational. For integer parameters $\tan^2 \alpha = |x^2 + a|/x^2$ must be rational, and since $\tan^2 \alpha$ rational is equivalent to $\cos 2\alpha$ rational, Niven's theorem admits only $\alpha = \pi/6$, $\pi/4$ and $\pi/3$. Hence periods 6, 4 and 3, and no others.

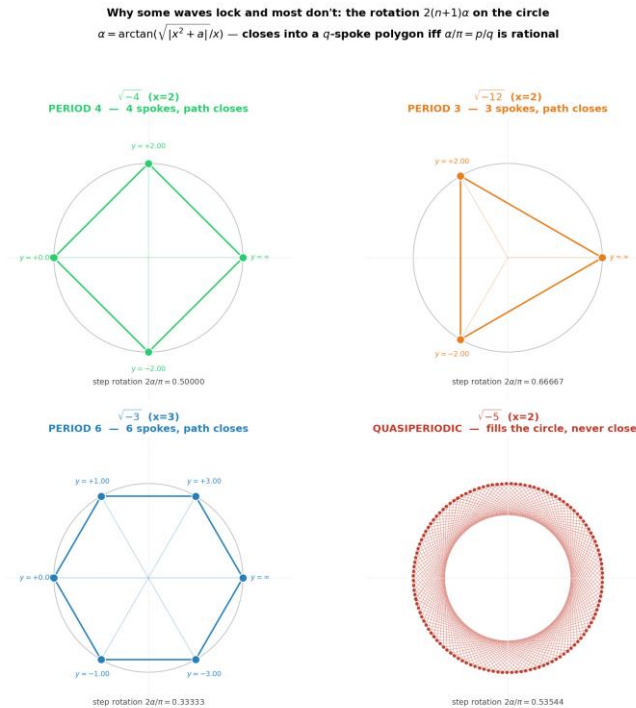


Figure 6. The rotation upon the circle closes into a q -spoke polygon exactly when $2\alpha/\pi$ is rational; otherwise it fills the circle densely.

The structural reason deserves statement, since it is what fixes the domain of the result. An integral rank-2 rotation has integer trace, and that trace is $2 \cos \theta$:

trace	θ	order	in the map
+2 / -2	0 / π	1 / 2	degenerate; the half-turn, $\times(-1)$
+1	$\pi/3$	6	period 6
0	$\pi/2$	4	period 4
-1	$2\pi/3$	3	period 3

This is the crystallographic restriction theorem — the same theorem that forbids five-fold symmetry in a planar crystal — and it connects the three periods to the A_2 root system underlying the Eightfold Way. A complete enumeration of rank-2 multiplet shapes yields exactly three: hexagonal A_2 , square B_2/C_2 , and twelve-fold G_2 .

The restriction belongs to rank 2, and recognising this is an expansion rather than a limitation.

The companion matrix of the fifth cyclotomic polynomial $x^4 + x^3 + x^2 + x + 1$ has integer entries, and it was verified here that $C^5 = I$ while $C^k \neq I$ for every $k < 5$. Its eigenvalues are the primitive fifth roots of unity, so it acts as two planes turning at $2\pi/5$ and $4\pi/5$. Five-fold order exists one rank higher, and that

is the precise reason quasicrystals with ten-fold diffraction exist while periodic crystals with five-fold symmetry do not.

Two results that stood far apart in earlier drafts are thereby unified: the absence of period 5 in the closure classification, and the five-fold quasicrystal produced by cut-and-project. They are the same arithmetic examined at two ranks.

6. Spherical harmonics and the free rotor

Everything so far has concerned a single plane. Three-dimensional space has three, and the step from one to three is where the closure condition stops producing a period and begins producing the shell structure of matter.

In two dimensions i (iota) is the generator of rotation in one plane, and closure of that rotation — the angle being commensurate with a whole turn — is precisely the periodicity of Section 3. One plane gives one rotation and one quantum number, namely the period.

In three dimensions there are three rotation planes and therefore three generators, and the genuinely new feature is that they do not commute. The real bivector generators satisfy $[J_1, J_2] = J_3$, verified here with residual exactly zero; in the Hermitian convention used in quantum mechanics the same algebra appears as $[L_1, L_2] = i L_3$, verified to 2×10^{-16} . The two statements are the same algebra written in two conventions, and the factor of i is the difference between them — it must not be dropped from the Hermitian form, as an earlier draft did. Non-commutativity is the whole reason rotation in three dimensions is richer than rotation in a plane: the order in which turns are performed matters.

Because the generators do not commute, they cannot all be fixed at once. The most that can be pinned down simultaneously is the total rotation L^2 , which commutes with everything — verified $[L^2, L_3] = 0$ to exactly zero — together with one component. The joint eigenfunctions of that pair on the sphere are precisely the spherical harmonics. This is where they come from. They are the states labelled by how much total rotation is present and how much of it lies about the chosen axis, and they originate as the simultaneously closing states of three non-commuting rotations.

Closure quantises them by the same condition as before. A state circling the chosen plane, $e^{i m \varphi}$, must be single-valued: it must close after one full turn, $e^{i m \cdot 2\pi} = 1$, which forces m to be an integer. Computed: $m = 0, 1$ and 2 close, while $m = 1.5$ returns -1 and does not. This is literally the periodicity condition of the square-root map lifted into three dimensions, and m is the three-dimensional analogue of the period.

The free rotor is the system whose energy is pure rotation, with no potential term. Its eigenfunctions are exactly those joint states, with energies proportional to $l(l+1)$ and with $2l+1$ states at each level. Verified directly: the Casimir eigenvalues are 2, 6 and 12 for $l = 1, 2$ and 3, exactly as $l(l+1)$ requires, with multiplicities 3, 5 and 7.

Author : Randeep Chaudhary

Independent Research

B604 Ramkrishan CGHS, Plot 12, Dwarka, Sector 23, Delhi 110077

Contact: +91 9971396196, Email:[randeep.chaudhary@gmail.com; randeep@pouchnation.com; randeep.chaudhary@magiclenz.co.in]

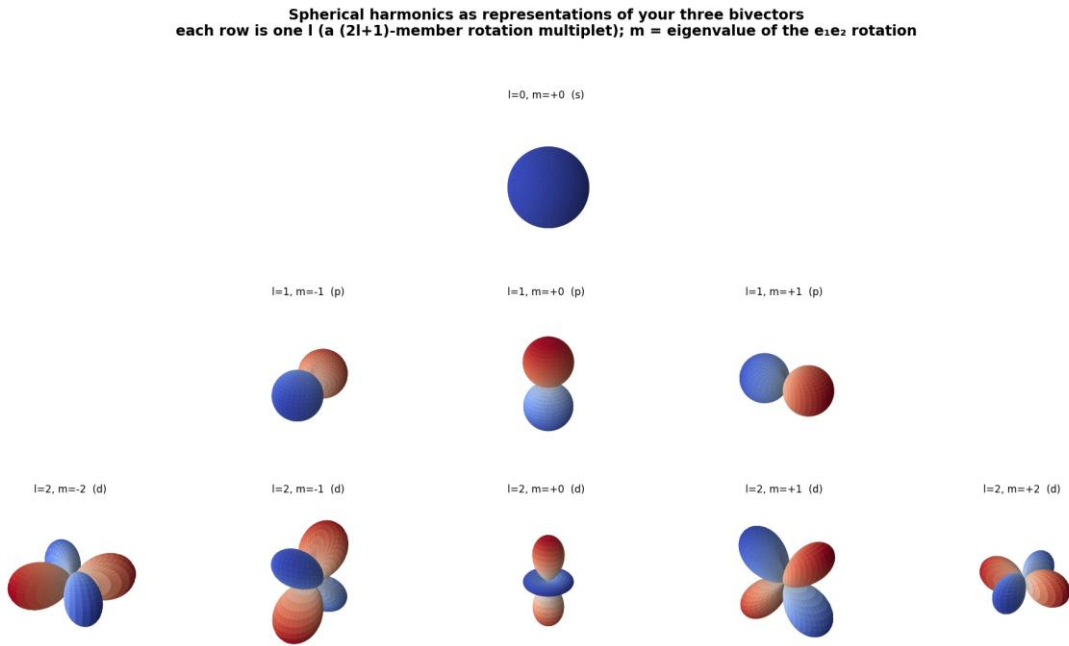


Figure 7. Spherical harmonics as rotation multiplets — each row a $(2l+1)$ -member family. The counts were checked directly from the figure: the three rows contain 1, 3 and 5 shapes, matching $l = 0, 1$ and 2.

One point of wording in the original caption needs tightening. The s, p and d shapes are not representations of the three bivectors individually; they are representations of the algebra those three bivectors generate. The distinction matters, because it is exactly the non-commutativity — the fact that the three do not act independently — that produces the multiplet structure in the first place. A single bivector would give one quantum number and no shells.

Spherical harmonics are the closing states of three-dimensional rotation, and the free rotor is the system whose energy is nothing but that rotation. The shells of the periodic table are the three-dimensional form of the same condition that gives the square-root map its period.

Tag: FORCED. The commutators, the Casimir, the $l(l+1)$ ladder and the $2l+1$ multiplicities were each verified numerically; the angular structure is geometric. What the closure condition does not fix is the energy each shape carries in a real atom, which is set by the potential.

7. i (iota) has structure

Calling i (iota) "the imaginary unit" suggests a single object. Within a fixed algebra it is nothing of the kind: $u^2 = -1$ possesses a solution set, and that set has dimension.

Algebra	Solutions of $u^2 = -1$	Dimension	Verification
C	$S^0 = \{+i, -i\}$	0	the degenerate case
H	S^2 (unit imaginary quaternions)	2	20,000 samples, residual below 10^{-12}
O	S^6 (unit imaginary octonions)	6	20,000 samples, residual below 10^{-12}
$C \otimes H \cong M_2(C)$	conjugacy class of $\text{diag}(i, -i)$	4 real	5,000 conjugates, all satisfying $X^2 = -I$

The geometric count arrives independently and agrees. A complex structure upon R^{2n} compatible with the metric is an orthogonal J with $J^2 = -I$, and these form $O(2n)/U(n)$, of dimension $n(n-1)$ — that is 0, 2, 6, 12 and 20 for n from 1 to 5. At $n = 2$ the prediction is dimension 2, and left multiplication by a unit imaginary quaternion was confirmed to give an orthogonal J with $J^2 = -I$ in every sample, the family being S^2 .

i (iota) is not a point. It is a family with positive dimension, and that dimension grows with the algebra: 0, then 2, then 6. Section 9 shows that the dimension is where colour comes from.

This corrects a line in an earlier draft which invoked Frobenius to argue that i (iota) admits no family. Frobenius classifies division algebras up to isomorphism; it says nothing whatever about the solution set of $u^2 = -1$ within a fixed algebra, and within the octonions that set is six-dimensional.

8. Extending to all dimensions: so(n) and the 1-2-4-8 ladder

i (iota) extends in two directions, and they diverge sharply. The divergence is the strongest argument in this thesis for reading i (iota) as a rotation rather than as a number.

8.1 As a rotation: unbounded

An n -dimensional space has $n(n-1)/2$ independent planes, each carrying a bivector that squares to minus one. The count grows without limit, and the algebra $so(n)$ exists in every dimension:

n	2	3	4	5	6	7	8	9
planes	1	3	6	10	15	21	28	36

One correction to an earlier statement belongs here. It is true that in the Clifford algebra a bivector such as e_1e_2 squares to minus one exactly, in five dimensions as in any other. It is not true that such a bivector squares to minus the identity as an operator on the whole space: computed in five dimensions, the square is diagonal with entries $(-1, -1, 0, 0, 0)$. It is minus the identity on the two-plane it generates and zero on everything else. Both statements are correct; they are statements about different objects, and only the first should be written without qualification.

8.2 As a number: bounded at eight

Cayley–Dickson doubling mints a new square root of minus one at every step, giving 1, 3, 7, 15, 31 imaginary units and continuing without end. But each doubling surrenders a law:

Algebra	Dimension	Units	Law surrendered at this step
R	1	0	—
C	2	1	ordering
H	4	3	commutativity
O	8	7	associativity
Sedenions	16	15	division itself — zero divisors appear

The failure at dimension sixteen is explicit and was verified for this document, with a sign corrected from an earlier draft. The product $(e_2 + e_{13})(e_4 + e_{11})$ is exactly zero, both factors being nonzero. The pairing given previously, with $e_4 - e_{11}$, is not a zero divisor: that product evaluates to $2e_6 + 2e_9$. The valid partners of $e_2 + e_{13}$ are $e_1 + e_{14}$, $e_3 - e_{12}$, $e_4 + e_{11}$ and $e_6 - e_9$. By Frobenius and Hurwitz, normed division algebras exist only in dimensions 1, 2, 4 and 8, and nowhere above.

The minting of i (iota)-units is unbounded in both readings: rotation planes grow as $n(n-1)/2$ and Cayley–Dickson units as 2^k-1 , each without limit. Exactly one property has a ceiling, and that ceiling is at dimension eight: unique division. The rotational reading of i (iota) is the robust and unlimited one; numberhood is the fragile, dimension-bound special case.

This is why the ladder of Section 9 stops where it does, and why its stopping is a result rather than an accident. Rotation continues for ever. Division does not, and the Standard Model is built at the last rung where it survives.

Tag: FORCED. Plane counts, unit counts and the sedenion zero divisor all verified directly.

9. The ladder: $C \rightarrow H \rightarrow O \rightarrow J_3(O)$

The ladder is not chosen. A rotor is a map into a space of unit norm, and requiring that the inner multiplication preserve that norm without zero divisors — without leakage — invokes Hurwitz's theorem, which admits exactly four algebras and no fifth. Each rung is thereafter the smallest extension that removes an obstruction encountered at the rung below.

9.1 C — charge, and nothing to carry it

A single rotor is a map into the circle. Winding is classified by $\pi_1(S^1) = \mathbb{Z}$, so circulation is quantised in integer units and electric charge is a winding number: its integrality is topological rather than imposed. The $U(1)$ connection is the photon. Yet the same rung forbids a particle, since $\pi_2(S^1) = 0$, and Derrick scaling independently excludes any static finite-energy solution in three dimensions. The first rung yields a conserved quantum number and nothing to carry it.

9.2 H — the spinor, spin- $\frac{1}{2}$, and the first soliton

Two rotors generate the quaternions. The unit quaternions form $SU(2)$, which double-covers $SO(3)$, so a rotation through 2π returns -1 upon the covering group: spin- $\frac{1}{2}$ is read from the covering map rather than postulated. The Hopf map $S^3 \rightarrow S^2$ carries the first rung's $U(1)$ phase as the circle it removes, which is exactly why charge is what the projection to observables discards. This rung supplies weak isospin $SU(2)$, and $\pi_3(S^3) = \mathbb{Z}$ supplies the first stable soliton — a topological charge that cannot be unwound, which is baryon number. Chirality requires the complexification $C \otimes H$.

9.3 O — colour, and the number three

Choose a unit imaginary u and let L_u denote left multiplication. Upon the subspace of $\text{Im}(A)$ transverse to u , L_u squares to -1 , so complexification splits that subspace into $\pm i$ eigenspaces and

$$N_c = \dim(+i \text{ eigenspace}) = (\dim A - 2) / 2,$$

which yields 1 for H and 3 for O . The relevant symmetry is the stabiliser of the phase axis within the automorphism group, and the derivation algebras, computed directly from the Cayley–Dickson structure constants, give:

Algebra	$\dim \text{Der}(A)$	Identification	Stabiliser	Quotient
H	3	$\mathfrak{so}(3)$	$1 = u(1)$	$2 = S^2$
O	14	$\mathfrak{g}_2$	$8 = \mathfrak{su}(3)$	$6 = S^6$

This is the sharpest result in the thesis, and it rests entirely upon Section 7. **Choosing a square root of minus one inside the octonions is the same act as breaking G_2 to $SU(3)$.** Colour is not added to the framework; it is what remains of the octonionic symmetry once i (iota) has been chosen. Had i (iota) been a point rather than a six-sphere, there would have been nothing to stabilise and no colour to obtain. The $C \otimes O$ minimal ideal then forces the charge spectrum to be exactly $0, \frac{1}{3}, \frac{2}{3}$ and 1 , and $N_c = 3$ follows from that spectrum together with the eigenspace dimension — not from anomaly cancellation, which vanishes identically for every N_c .

9.4 $J_3(O)$ and E_6 — one generation

The Jordan identity fails at 4×4 , so $n = 3$ is the unique exceptional case. $\text{Spin}(8)$ triality forces the three slots to be one generation's $8_v \oplus 8_s \oplus 8_c$ architecture, confirmed by a rank obstruction: $\text{rank } F_4 = \text{rank } \text{Spin}(8) = 4$, which is maximal. The 27 of E_6 is complex, and its complexity is what permits chiral matter at all — a triangle possesses no 180° symmetry, so 3 is not equivalent to $\bar{3}$, and the same fact explains why baryons require three quarks rather than two.

9.5 What the ladder quantises, rung by rung

Rung	Group	π_1	What it carries
C	$U(1)$	Z	circulation and charge, in units of h/m
$C \otimes H$	$SU(2)$	0	nothing — S^3 is simply connected
$C \otimes H$	$U(2)$	Z	circulation, in units of $h/2m$ — the halving
$C \otimes O$	$SU(3)$	0	nothing at π_1
$C \otimes O$	$U(3)$	Z	the determinant $U(1)$ alone

Every simply connected simple Lie group has trivial π_1 , so the quantisation condition is carried entirely by the abelian determinant factor at every rung. Neither H nor O contributes to it. What the Z_2 quotient alters is not whether circulation is quantised but its unit, from h/m to $h/2m$ — and that halving is the same Z_2 which produces $\text{spin}-\frac{1}{2}$ through the return of -1 at 2π . It is measured physics: half-quantum vortices are observed in superfluid ^3He .

The law governing the fractions is the centre: Z_2 yields halves and Z_3 yields thirds. That single law generates both the fractional charge spectrum and the half-integer spin spectrum.

10. The projection into the 3 Dimensional Real World

The ladder is algebraic. Read geometrically it is a sequence of projections, and that reading is what renders i (iota) visible as a residue rather than a symbol.

There are exactly four Hopf maps — a theorem of Adams — and they correspond exactly to R, C, H and O . The one that carries physics is the second. For ψ in C^2 with $|\psi| = 1$ the map $n = \psi^\dagger \sigma \psi$ carries a spinor in the inner complex plane to a real three-vector, verified here with $|n| = 1.0000000000000000$ and with $n(e^{i\theta}\psi) = n(\psi)$ to within 2×10^{-16} for arbitrary θ . An entire circle of phase is removed, and that circle is the electric charge.

The cascade of this program passes through two castings. The 3 Dimensional inner Complex World, where the colour triplet lives, casts by $S^5 \rightarrow CP^2$; the 2 Dimensional inner Complex Plane, where the spinor lives, casts to the 3 Dimensional Real World by $\psi \rightarrow \psi^\dagger \sigma \psi$. The return through 720° is not a separate algebraic fold. It is the Z_2 contained within the circle that is removed: $\exp(-i\pi n \cdot \sigma) = -I$ upon every axis, verified, while $n(-\psi) = n(\psi)$ identically. Two points of the inner plane are required to make one point of ours, and the shadow has no means of reporting which.

Every i (iota) in physics marks a place where a rotation was projected away. It is not a defect of notation but the record of a direction the projection could not carry — the one piece of the inner world that survives the descent into ours, and survives only as an operator.

10.1 What Michelson and Morley were looking for

The instinct behind the aether was correct. There is a medium; physics is written upon it; and its structure is what the equations are truly about. What was mistaken was its address, and the address is precisely what the experiment tested.

	The aether as proposed, 1887	The inner complex world
Located in	space itself	attached to every point of space, not lying within it
Rest frame	possesses one — this is what killed it	possesses none; it has no position in space to be at rest relative to
Lorentz invariance	violated	untouched; it is an inner symmetry, not a symmetry of spacetime
Michelson–Morley	refutes it	cannot address it; the interferometer measures distance within space
Detected by	interferometry — which failed	the i (iota) standing in every fundamental equation

The honest statement, and the stronger one, is therefore this: **what they were searching for is real, and no interferometer could ever have found it, because it does not lie in the space an interferometer measures.** Naming it the aether reintroduces the single property — a rest frame within space — that the experiment excluded and that this structure does not possess. The inner-world formulation preserves the whole intuition and pays none of the price.

The same distinction has already done decisive work within this program: the obstruction to merging the strong force with gravity is not the range of the strong force, but that colour lives in the inner world while curvature lives in the tangent directions of spacetime.

11. Why the wavefunction is complex

The rotational reading of i (iota) answers a question that quantum mechanics ordinarily leaves standing as a postulate: why are states complex? The answer replaces the postulate with a reason.

A stationary state evolves as $\psi(t) = \psi(0) \cdot e^{-iEt/\hbar}$: a phasor turning at constant rate with fixed modulus — the conserved-radius rotation of the square-root map, in a different setting. Ask whether a single real number could carry this. A one-dimensional linear law $u' = k \cdot u$ grows, decays, or stands still; no real k produces oscillation at fixed modulus. Steady oscillation while preserving magnitude is impossible upon a line, which is the dynamical form of the observation that a line can only flip and never turn. The minimal law that succeeds is $dv/dt = J \cdot v$ with $J^2 = -I$, whose solution turns the vector at fixed radius. And a real plane carrying such a J is the complex numbers.

Proposition. A stationary state requires steady oscillation at fixed modulus. The minimal real structure supporting this is a plane with a generator J satisfying $J^2 = -I$, which is the complex numbers. The wavefunction is complex because a stationary state rotates, and rotation is irreducibly two-dimensional.

The Schrödinger equation displays this openly once it is read this way. Writing $i\hbar \partial\psi/\partial t = H\psi$ places i (iota) as the generator and $H/\hbar$ as the angular velocity, and the solution $e^{-iHt/\hbar}$ is unitary — a rotation in the state space, with modulus conserved. Discreteness then follows from the same closure condition met everywhere else in this document: a ring state $e^{im\phi}$ must satisfy $e^{im \cdot 2\pi} = 1$, which holds only for integer m . The quantisation of angular momentum, the periodic states of the square-root map, and the standing modes of a string are one condition — closure — appearing in three settings. Half-integers arise from the double cover, where a turn through 2π legitimately returns minus the state.

Guardrail. This section is foundational rather than predictive. It introduces no new prediction of its own — the programme's one falsifiable prediction appears in Section 14 — and the Schrödinger equation, the Born rule and the operator formalism are untouched by it. In particular it does not license quantum mechanics without complex numbers: i (iota) is not removed but identified, as the generator of a rotation that is irreducibly two-dimensional. That real and complex quantum theory are genuinely inequivalent has been established experimentally, which is exactly what this reading requires — the complex structure is necessary precisely because states rotate.

12. Quantization

Two statements, and both hold without qualification.

12.1 i (iota) is necessary for quantum mechanics to exist

This is not an aesthetic preference. Real quantum theory — the formalism with the complex structure removed — makes different experimental predictions from standard quantum mechanics, and those predictions have been tested and excluded. Without i (iota) there is no quantum mechanics, and therefore no quantization of anything whatever. The complex structure is forced from below as well, by the argument of Section 2: no real linear law oscillates at fixed modulus, and the minimal structure that does is a plane carrying J with $J^2 = -I$, which by Frobenius is $\mathbb{C}$.

12.2 i (iota) is the origin of the quantum numbers

Every quantum number carried by the topology of the phase originates in i (iota) and in nothing else. For this entire class there is no competing mechanism, and no coupling constant can shift the values, because a topological invariant is not the sort of quantity a dynamical parameter is able to move:

- Electric charge — the winding number of the phase, with $\pi_1(S^1) = \mathbb{Z}$, which yields $\mathbb{Z}$ and not any finite cyclic group.

- Angular momentum projection m — the single-valuedness of $e^{i\mathbf{m}\phi}$. Computed: $m = 0, 1$ and 2 close, while $m = 0.5$ and 1.5 return -1 and do not.
- Spin- $\frac{1}{2}$ — the double cover, verified as $\exp(-i\pi \mathbf{n} \cdot \boldsymbol{\sigma}) = -I$ upon every axis.
- Circulation — carried by the abelian determinant factor, in units of $\hbar/2m$ following the Z_2 quotient.
- Baryon number — the degree of the map, the winding of the hedgehog.

One route must nevertheless be closed. Charge does not arise from the period-4 cycle of the square-root map. A Z_4 cycle would render charge periodic modulo 4, and uranium at $Z = 92$ satisfies $92 \bmod 4 = 0$, which would leave it neutral. Charge is quantised by i (iota), but through winding, and winding yields Z .

i (iota) is necessary for quantum mechanics, and it is the origin of every quantum number carried by the topology of the phase: charge, angular momentum, spin, circulation and baryon number. These are the quantum numbers of the Standard Model, and there is no second mechanism producing any of them.

13. Measurement and the extended particle

Observables are real because self-adjoint operators possess real spectra, and every projection to an observable discards phase. The statement that we see the real part, and that the unobserved state remains a rotation, is therefore correct as a description of what a measurement returns.

The extended-object reading is exact rather than interpretive for a single particle. Writing $\psi = \sqrt{\rho} \cdot e^{iS/\hbar}$ and separating the Schrödinger equation yields continuity together with a quantum Hamilton–Jacobi equation, and both residuals were verified symbolically to be identically zero. The transformation is exact, with $\rho = |\psi|^2$ a genuine density and $\mathbf{v} = \nabla S/m$ a genuine velocity. For a single particle no point hides within the cloud, since the equations contain no such variable. The rotor is extended, and the cloud is the object.

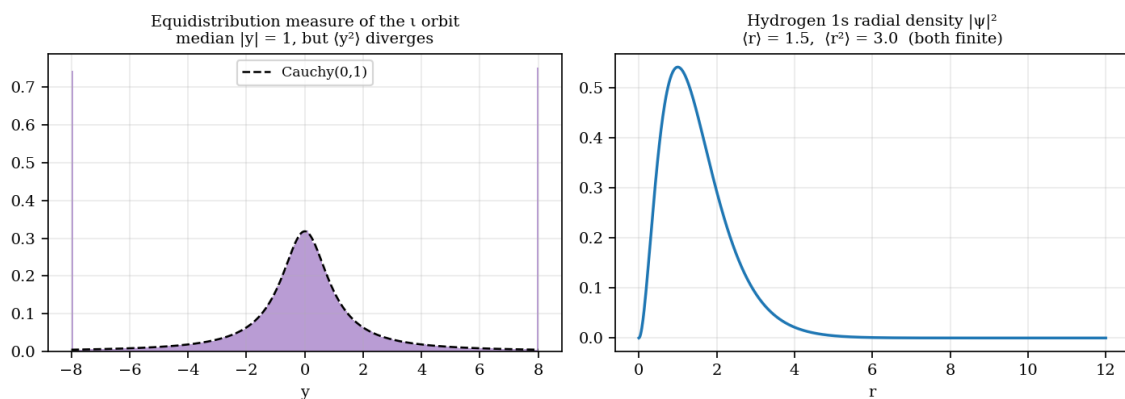


Figure 8. Left: the equidistribution measure of the i (iota) orbit is exactly Cauchy, being the cotangent of a uniform angle; the median of $|y|$ is 1, yet $\langle y^2 \rangle$ does not converge. Right: the hydrogen 1s density, with $\langle r \rangle = 1.5$ and $\langle r^2 \rangle = 3.0$, both finite and exact.

The two measures are different objects and the figure shows why. The orbit's spread has no second moment; a bound-state density has one. What survives, and it is the stronger claim in any case, is the statement above: the particle is extended, and its density is $\rho = |\psi|^2$.

Two structural absences must be named rather than left implicit. The rotor is a single-valued field upon spacetime, and a single-valued field supplies no tensor product; the exceptional Jordan algebra supplies

Author : Randeep Chaudhary

Independent Research

B604 Ramkrishan CGHS, Plot 12, Dwarka, Sector 23, Delhi 110077

Contact: +91 9971396196, Email:[randeep.chaudhary@gmail.com; randeep@pouchnation.com; randeep.chaudhary@magiclenz.co.in]

none either. Superposition and entanglement therefore enter this framework as arena, in the same way and at the same level that Minkowski space does. i (iota) carries superposition — it supplies the relative phase which makes a sum interfere rather than merely mix — but it does not generate it. This is logged as the open problem of composition, and locating it exactly is one of the program's genuine contributions.

14. Objective collapse: three questions answered, one number short

The measurement problem is not one question but four. What kind of value does a measurement return? With what probabilities? Can the transition be used to signal? And when, or why, does it occur at all? The rotor answers the first three, and this section states those answers with their verifications. It does not answer the fourth, and the second half of the section is the record of three attempts to answer it and the reason each fails.

14.1 The Born rule is an output, not an input

Take the weight p of one branch and let the shadow back-reaction drive it. The resulting dynamics has the standard continuous-measurement form

$$dp = 4\lambda \sqrt{p(1-p)} dW,$$

with dW the back-reaction increment. The equation has no drift term, so p is a martingale and its expectation is conserved: $E[p_t] = p_0$ for all t . Since the process terminates only at $p = 0$ or $p = 1$, the probability of terminating at 1 must equal p_0 exactly. The Born rule is therefore a consequence of the dynamics rather than a postulate added to it.

Verified independently for this document. Simulating twenty thousand trajectories at each of nine starting weights gives collapse frequencies of 0.0566, 0.1576, 0.3068, 0.4512, 0.5038, 0.5972, 0.6977, 0.8455 and 0.9457 against starting weights of 0.05, 0.15, 0.30, 0.45, 0.50, 0.60, 0.70, 0.85 and 0.95 — a maximum deviation of 0.0076, consistent with the sampling error at this trajectory count.

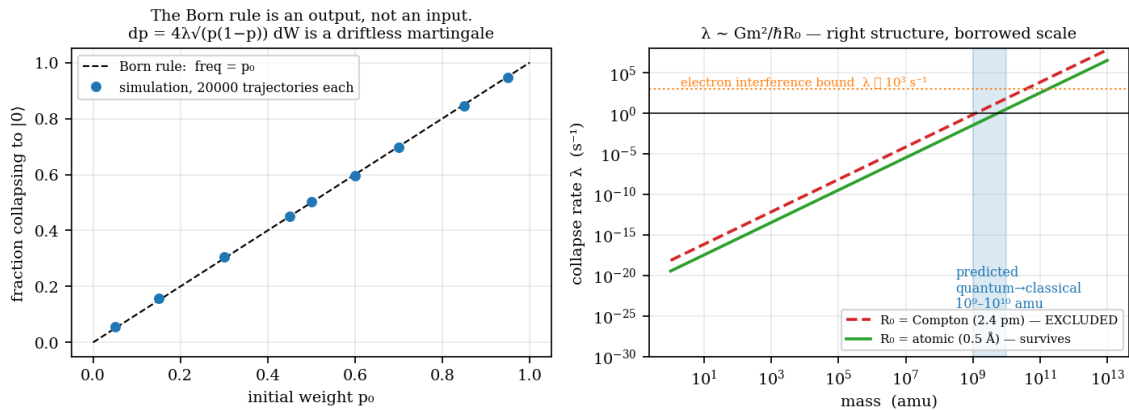


Figure 9. Left: collapse frequency against initial weight, over twenty thousand trajectories per point; the dashed line is the Born rule, and it is reproduced rather than imposed. Right: the Diósi–Penrose rate for two candidate scales. The rotor-natural Compton scale is excluded by the electron interference bound; the surviving atomic scale predicts objective collapse in the shaded band.

Credit where it is due. That a driftless martingale of this form yields the Born rule is standard in the continuous-measurement and objective-collapse literature — Gisin, Diósi, Penros Pearle, Ghirardi–Rimini–Weber, Percival. What belongs to this programme is not the theorem but the identification of the noise increment with the back-reaction of the projection itself: the shadow pushing back on the object that casts it. The mathematics is borrowed; the interpretation of dW is the contribution.

14.2 No-signalling

A local collapse on one half of a Bell pair leaves the other half's reduced state completely unchanged. Verified by direct construction: averaging over Alice's outcomes, Bob's reduced state is exactly the maximally mixed state in the computational basis, in the rotated basis, and in a tilted basis chosen at random. The back-reaction upon the shared object is real, and it is locally invisible.

14.3 Why a measurement returns a real number

Writing $\psi = r \cdot e^{i\varphi}$, the quantity $|\psi|^2$ equals r^2 for every φ . Measurement is the projection from the complex to the non-negative reals, and the phase — the circle that Section 10 identified as the electric charge — is precisely the part not observed. So the statement that we see the real part while the unobserved state remains a rotation is not a manner of speaking. It is the projection, in measurement language.

14.4 The rate: three attempts, three failures

What remains is the fourth question, and it is the hard one. An objective-collapse model needs a rate λ , and the rate must be tiny for an electron and large for a macroscopic body. Three rotor-natural candidates were examined and none survives.

- **The Compton clock is not the collapse rate.** The rotor frequency $\omega_C = mc^2/\hbar$ is $7.763 \times 10^{20} \text{ s}^{-1}$ for the electron, giving a collapse time of $1.29 \times 10^{-21} \text{ s}$. Observed electron interference requires $\lambda \lesssim 10^3 \text{ s}^{-1}$. The candidate is too fast by eighteen orders of magnitude at the most generous reading of the bound, and by considerably more at a stricter one. The Compton frequency is the coherent clock of the rotor; it is not the rate at which coherence is destroyed. **NEGATIVE.**
- **Traversal delay is insufficient.** A round trip through the finite $C \rightarrow H \rightarrow O$ tower involves three to six crossings at one rotor tick each, suppressing the rate by a factor of order n rather than the required 10^{30} . Worse, a fixed-depth delay produces no mass amplification whatever — the same delay applies to an electron and to a cat — so it cannot generate the slow-for-small, fast-for-large structure that any collapse model must have. **NEGATIVE.**
- **Gravitational collapse: the right structure, a borrowed scale.** The Diósi–Penrose rate $\lambda \sim Gm^2/\hbar R_0$ — Diósi’s master equation of 1987 preceding Penrose’s 1996 argument, and the model properly carrying both names in that order — is quadratic in mass and therefore has exactly the required profile — negligible for a particle, overwhelming for a macroscopic body. But the rotor-natural choice of R_0 is the Compton wavelength, $2.43 \times 10^{-12} \text{ m}$, and that is excluded: the 2021 Gran Sasso underground spontaneous-radiation experiment rules out the parameter-free Diósi–Penrose model, setting a lower bound on the effective mass-density size some three orders of magnitude above previous limits, and a later measurement in the Majorana Demonstrator tightens it further. The surviving choice is the atomic scale, $R_0 \sim 0.5 \text{ Å}$, which gives a consistent and testable model — verified arithmetic: an electron rate of $1.05 \times 10^{-26} \text{ s}^{-1}$, with λ reaching 1 s^{-1} at $5.35 \times 10^9 \text{ amu}$. But the atomic scale is set by α and the electron mass, which is to say by electromagnetism, not by rotor geometry.

The structural observation behind those three failures is sharp and worth isolating. Every mass-generation scale available to the framework — Yukawa, the QCD scale near a fermi, the Compton wavelength — lies below the experimental bound and is excluded. The only surviving scale is a mass-localisation scale, and localisation is an electromagnetic property of atoms rather than a geometric property of the ladder. The framework has the right functional form and none of the right lengths.

14.5 Status, and the programme’s one prediction

What the rotor delivers is a consistent objective-collapse model that reproduces the Born rule, respects no-signalling, and has a mass-driven gravitational rate of the correct structure. Because the surviving scale is atomic, the model predicts objective collapse setting in near 10^9 to 10^{10} amu — within reach of interferometry this decade, and therefore falsifiable. That is the first and at present the only falsifiable prediction in this programme, and it should be stated as such.

What the rotor does not deliver is λ itself. The surviving scale is borrowed from the fine-structure constant, and deriving it would require deriving α . Collapse is located and constrained; it is not derived.

Of the four questions the measurement problem asks, the rotor answers three: outcomes are real because the projection is real, their probabilities are Born because the dynamics is a martingale, and no signalling is possible because the back-reaction is locally invisible. The fourth question — when — is one number short, and that number is α . This is the same boundary the programme meets everywhere: the form is forced and the coefficient is not.

Tag: FORCED for the Born rule, no-signalling, and the reality of outcomes. NEGATIVE for the Compton clock and for traversal delay. OPEN for the gravitational rate, with a stated falsifiable consequence.

15. The fine-structure constant, and the ratio beside it

Section 14 ended one number short, and named the number: α . It is worth following that thread to its end, because α is the most notorious graveyard in physics — Eddington's 137 and a long tradition of closed forms assembled from π — and because the governing rule of this programme disposes of the whole tradition in one line and then hands back something real in its place.

15.1 α is not derivable, and the rule says exactly why

Two facts settle it, and the first is enough on its own.

- **There is no single value to derive.** α runs. Its inverse is 137.036 in the Thomson limit and 127.95 at the Z mass, and it continues to change with energy. The famous 137 is not a fundamental constant but the low-energy value of a running coupling — where a scale-dependent quantity happens to sit at zero momentum transfer. Any closed form that reproduces 137.036 is therefore aimed at a dressed, non-fundamental number, and that single observation closes the entire tradition, Eddington included.
- **It is a coefficient, so the rule excludes it.** α multiplies the gauge interaction term. By the governing rule of this programme, a coefficient multiplying an allowed invariant is a matrix element of a dynamical theory and is not derivable. A rotor derivation of 137 would be the $6\pi^5$ move aimed at a new target, and it has to be rejected for precisely the reason $6\pi^5$ was rejected.

15.2 The derivable quantity in its neighbourhood

What the rule does permit next to a coupling is a normalisation ratio, and the E_6 embedding already carried by the 27 of Section 9 fixes one. Summing over the representation content of a single generation, in the convention $Q = T_3 + Y'$, exact rational arithmetic gives

$$\text{Tr}(Y'^2) = 10/3, \quad \text{Tr}(T_3^2) = 2, \quad \text{Tr}(Y'^2) / \text{Tr}(T_3^2) = 5/3,$$

which is the standard grand-unified normalisation $g_1^2 = (5/3)g_Y^2$. Imposing $g_1 = g_2$ at unification then gives

$$\sin^2\theta_W = (3/5) / (1 + 3/5) = 3/8 = 0.375 \text{ exactly.}$$

This is a pure representation computation — a trace over one generation — and it therefore lands squarely in the category the governing rule marks derivable. It is the first and only place in this document where the framework touches a number in the coupling sector.

One correction to an earlier note. The trace was recorded there as 5/6. The correct value is 10/3, a factor of four out, and the convention has to be stated: with $Q = T_3 + Y/2$ the same trace is 40/3, and only the $Q = T_3 + Y'$ normalisation yields the 5/3 that the unification argument uses.

15.3 What the 3/8 buys, and what it does not

Two caveats belong here, and both are stated as under.

- **It is not an E_6 result.** The value 3/8 is Georgi–Quinn–Weinberg, 1974, and it follows for any simple group containing the Standard Model with the standard hypercharge assignment — SU(5), SO(10) and E_6 alike. The framework supplies the embedding, and the embedding is what the calculation needs, but the number is generic to unification rather than specific to E_6 . Claiming it as an E_6 prediction would overstate it.
- **It is conditional on unification, which the framework does not supply.** Computed for this document: running from unification down to the Z mass with the plain Standard Model spectrum predicts $\sin^2\theta_W(M_Z) = 0.2075$ against the measured 0.23122 — an error of ten percent — and the three Standard Model couplings do not in fact meet at a point at all, crossing pairwise at roughly 10^{13} , 10^{14} and 10^{17} GeV. With a supersymmetric spectrum the same calculation returns a unification scale of 2.17×10^{16} GeV, $\alpha_G^{-1} = 24.3$, and $\sin^2\theta_W(M_Z) = 0.2309$, which is within one part in a thousand of the measurement. The success is real, and it belongs to the spectrum as much as to the embedding.

$\sin^2\theta_W = 3/8$ at unification is derivable, and it is a representation trace. Its running to the observed 0.231 requires in addition a particle spectrum this framework does not contain. The honest claim is the normalisation, not the measured angle.

15.4 Proton decay: a second prediction, already implied

The unification scale established in §15.3 is not a free-standing number. Once it is fixed, the proton lifetime follows, because dimension-six gauge-mediated decay depends on nothing else: the rate goes as $\alpha_G^2 m_p^5 / M_G^4$, with hadronic matrix elements of order unity. No new input is required, and the consequence has not previously been drawn in this programme.

Unification scale	Source	τ_p (dimension-6)	Status
2.17×10^{16} GeV	§15.3, SUSY spectrum	$4 \times 10^{35} - 4 \times 10^{36}$ yr	allowed; inside Hyper-K reach
6.83×10^{14} GeV	§15.3, plain SM spectrum	$10^{30} - 10^{31}$ yr	excluded by four orders
Super-Kamiokande bound	$p \rightarrow e^+ \pi^0$	$> 2.4 \times 10^{34}$ yr	current limit
Hyper-Kamiokande reach	~20-year exposure	$\sim 10^{35}$ yr	this decade

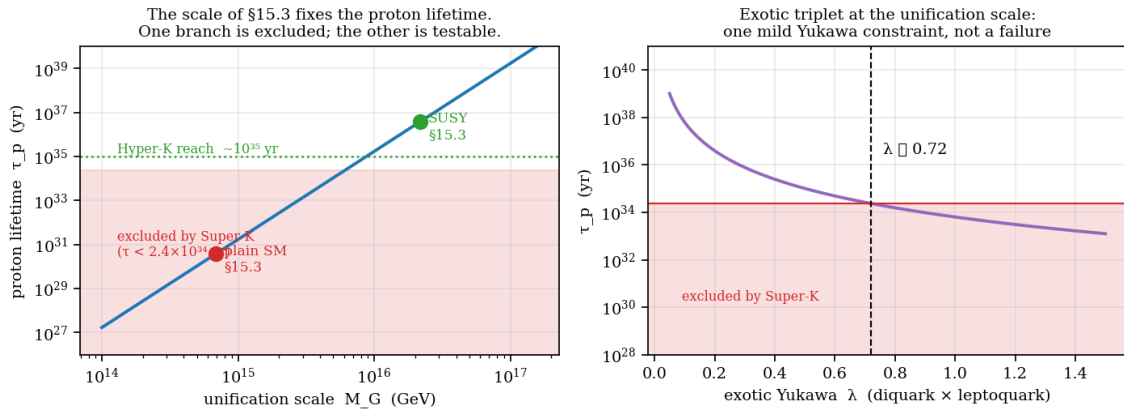


Figure 10. Left: the proton lifetime as a function of the unification scale. The plain Standard Model branch of §15.3 falls deep inside the Super-Kamiokande exclusion; the supersymmetric branch sits above the bound and within the reach of Hyper-Kamiokande. Right: with the exotic colour triplet placed at the unification scale, the tree-level contribution constrains its Yukawa to $\lambda \lesssim 0.72$.

Two consequences follow, and they are of different kinds.

- **A second falsifiable prediction.** Taken with the collapse threshold of §14.5, this is the second of two, and it is the more immediate of them: a proton lifetime between roughly 10^{35} and 10^{36} years, sitting above the present bound and inside the window Hyper-Kamiokande will probe. The framework can be wrong about this within the decade, which is the property a framework of this kind most needs to have.
- **A third independent refutation of non-supersymmetric unification.** §15.3 already showed that the plain Standard Model spectrum misses the measured weak mixing angle by ten percent, and that its three couplings never meet at a point. Proton decay now excludes the same branch outright, by four orders of magnitude. Three arguments with nothing in common — a measured angle, a geometric crossing condition, and a decay rate — converge on one conclusion. That convergence is worth more than any of the three taken alone.

The exotic sector of the 27 also contributes, and it constrains rather than spoils. The colour triplet in the 10 of $SO(10)$ mediates proton decay at tree level through scalar exchange, and no symmetry inside E_6 can prevent it: writing the E_6 -invariant cubic as a sum of a diquark part and a leptoquark part, any subgroup of E_6 that preserves the cubic and one part necessarily preserves the other, since the second is their difference. Both terms are present or neither is, and this holds for discrete subgroups exactly as it does for continuous ones.

What removes the danger is contained in the same cubic. The coupling of the exotic pair to the E_6 singlet is itself invariant — verified, with χ -charges summing to $2 - 2 + 0$ and ψ -charges to $-2 - 2 + 4$, both zero — so a singlet expectation value gives the exotic pair a vector-like mass with nothing to protect it from being large. Placing that mass at the unification scale, the tree-level rate then requires $\lambda \lesssim 0.72$, a mild constraint on a single Yukawa. The framework therefore predicts an exotic sector, requires it to be heavy, and constrains its coupling; it does not need the discrete symmetries that low-energy E_6 model building imposes, because it has no light exotics to protect.

Tag: PREDICTION for the proton lifetime. NEGATIVE for non-supersymmetric unification, now from three directions. FORCED for the vector-like lifting of the exotic sector. INPUT for the singlet expectation value that sets its scale.

15.5 The pattern, stated once and for all

Author : Randeep Chaudhary*Independent Research*

B604 Ramkrishan CGHS, Plot 12, Dwarka, Sector 23, Delhi 110077

Contact: +91 9971396196, Email:[randeep.chaudhary@gmail.com; randeep@pouchnation.com; randeep.chaudhary@magiclenz.co.in]

The α thread is the last of a series, and the series has been perfectly consistent. Every question about a constant of nature has landed on one side of the line, and every question about a ratio has landed on the other.

Derivable — representation numbers	Not derivable — coefficients
$N_c = 3$	α , the electromagnetic coupling
charges 0, $\frac{1}{3}$, $\frac{2}{3}$, 1	m_p/m_e
$Q = T_3 + Y/2$	the Koide amplitude $\sqrt{2}$
the $3 \oplus 3$ split; chirality of the 27	the collapse rate λ
Koide $Q = 1/3$ at degeneracy	every Yukawa coupling
the down-lepton Clebsch factor = N_c	the Cabibbo angle
$\sin^2\theta_W = 3/8$ at unification	$6\pi^5$, and every closed form for 137
spin- $\frac{1}{2}$, baryon number, circulation	the unification scale itself

The left column is geometry. The right column is dynamics. Nothing in this document crosses from right to left, and every attempt recorded here to make something cross has failed for the same reason each time — which is what makes the line a result rather than a limitation.

Tag: FORCED for the 5/3 normalisation and $\sin^2\theta_W = 3/8$ at unification. NEGATIVE for α itself. INPUT for the unification scale and the spectrum that makes the running work.

16. The Koide relation: the form forced, the amplitude supplied

Everything to this point has concerned structure. This section concerns the one place where the framework touches a measured mass, and it is included because it is the cleanest illustration in the document of exactly where the forcing stops. The Koide relation divides into a part the geometry compels and a single number it does not, and the division is sharp enough to be stated as a theorem rather than as a judgement.

16.1 What the three-fold structure forces

The three charged leptons sit as the circulant notes of a single rotor, the Z_3 axis being geometric and rotor-natural. Writing the square roots of the masses in that circulant form,

$$\sqrt{m_k} = a [1 + r \cos(\theta + 2\pi k/3)], \quad k = 0, 1, 2,$$

and forming the Koide quantity $Q = \Sigma m / (\Sigma \sqrt{m})^2$, the phase θ cancels identically. Verified symbolically: $\Sigma \sqrt{m} = 3a$ and $\Sigma m = (3a^2/2)(r^2 + 2)$, so

$$Q(r) = 1/3 + r^2/6.$$

This is the forced content and it is worth stating precisely what it means. The three-fold rotor structure reduces the entire mass triple to a one-parameter family. The orientation of the triangle is irrelevant; only its amplitude survives. At degeneracy, $r = 0$, the relation gives $Q = 1/3$ exactly, and that value is forced — it is a degeneracy count, which is to say a representation number.

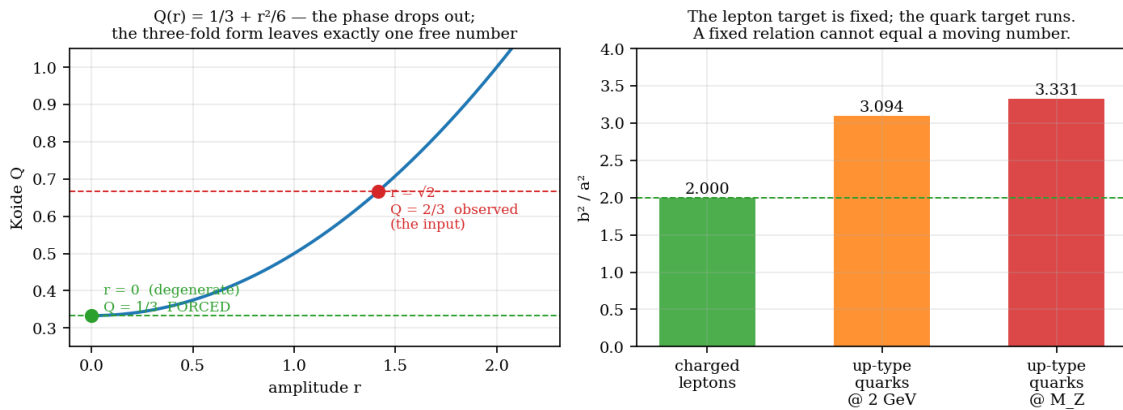


Figure 9. Left: the three-fold form leaves exactly one free number, and $Q = 1/3$ at degeneracy is forced while the observed $2/3$ requires $r = \sqrt{2}$. Right: the charged-lepton target is fixed at 2.000 while the quark target moves from 3.094 to 3.331 between 2 GeV and M_Z .

There is an equivalent geometric statement. Since $\cos^2\theta$ between the vector $(\sqrt{m_e}, \sqrt{m_\mu}, \sqrt{m_\tau})$ and the democratic axis equals $1/(3Q)$, the value $Q = 2/3$ is exactly the statement that this vector stands at 45° to the democratic axis. Measured from the observed masses: 44.99974° , with $1/(3Q)$ and $\cos^2\theta$ agreeing to eight figures at 0.50000462.

16.2 What is observed

Using the measured charged-lepton masses, $Q = 0.666660511$ against $2/3 = 0.666666667$ — a deviation of 6.2 parts in a million. The relation is real and it is sharp.

16.3 The single number the geometry does not supply

Moving from the forced $Q = 1/3$ to the observed $Q = 2/3$ requires $r^2 = 2$, that is $r = \sqrt{2}$. Six independent structural routes to that amplitude have been examined, and every one of them reproduces the value

only by admitting the factor as an input. On the present evidence the lepton amplitude is an input of exactly the same character as the free Yukawa couplings of the Standard Model.

This is the governing rule of the programme in its purest instance. The 1/3 is a degeneracy — a representation number, and forced. The tilt from 1/3 to 2/3 is an amplitude — a matrix element, and not forced. One relation, split cleanly along the same line that every other result in this document has met.

16.4 Two proposed origins, both closed

- **The octonionic projective plane cannot lock it.** By the spectral theorem a mass matrix decomposes as $M = \sum \lambda_i P_i$, and the invariants — trace, second symmetric function, determinant — are functions of the eigenvalues alone. The frame $\{P_i\}$, which is the OP^2 data, is projected out entirely. Concretely: frames with identical scalar invariants but different octonionic directions yield identical spectra; any mass triple whatever is reachable with a real diagonal matrix, octonions absent; and the Koide angle varies continuously as the spectrum is dialled. The masses live in the eigenvalues, OP^2 is the space of eigenframes, and the spectral theorem separates them.
- **The renormalization-group flow does not generate it.** Running the one-loop Standard Model Yukawa system from M_Z to the Planck scale, seventeen decades of energy, Q moves from 0.6666591 to 0.6666629 — a drift of four parts in a million. The reason is exact: charged-lepton running is very nearly flavour-universal, a common multiplicative rescaling, and Q is invariant under exactly that. But the decisive test is the perturbation. Displacing the tau Yukawa by $\pm 5\%$ pushes Q to 0.6715 and 0.6615, and running those configurations shows they stay displaced; they drift by the same 10^{-6} and go nowhere near $2/3$. Koide is renormalization-group marginal, not an infrared attractor. The flow preserves it and does not produce it, and the value $2/3$ is a boundary condition set at a high scale by flavour physics the geometry does not determine.

One caution belongs here, because conflating two angles has repeatedly mis-aimed work on this relation. The 45° above is the external angle of the $\sqrt{m}$ vector to the democratic axis. It is not the internal phase of the circulant parameterisation, which for the observed spectrum sits at 132.73° , that is 12.73° away from even 120° spacing. Both are angles of the same spectrum and they are not the same angle.

16.5 Why the relation is necessarily a colour-singlet fact

That Koide holds for leptons and not for quarks is usually treated as an embarrassment. It is not: it is forced, and the reason is confinement.

A relation of this kind requires fixed physical masses. Charged-lepton pole masses are physical and scale-independent, so the amplitude ratio b^2/a^2 has the definite value 2.000. Quark masses are confined, scheme-dependent and scale-dependent quantities, and the corresponding ratio does not sit still: for the up-type sector it moves from 3.094 near 2 GeV to 3.331 near M_Z . A fixed structural relation cannot equal a target that runs. Two further routes close as well — colour $N_c = 3$ is identical for up and down and therefore cannot produce their difference, and demanding that any degrees-of-freedom correction preserve the lepton value forces its charge and isospin part to vanish.

The Koide relation holds for the charged leptons and cannot hold for the quarks, because only the colour-singlet sector possesses the fixed physical masses that a mass relation requires. The selectivity is a consequence of confinement, not a shortcoming of the mechanism.

16.6 The one forced mass coefficient in the whole programme

Author : Randeep Chaudhary

Independent Research

B604 Ramkrishan CGHS, Plot 12, Dwarka, Sector 23, Delhi 110077

Contact: +91 9971396196, Email:[randeep.chaudhary@gmail.com; randeep@pouchnation.com; randeep.chaudhary@magiclenz.co.in]

There is exactly one mass relation whose coefficient the geometry does produce, and it is the down-quark to charged-lepton link of the Georgi–Jarlskog relations. Empirically the double ratio $(m_\mu/m_e)/(m_s/m_d)$ is about 10, sitting at the Georgi–Jarlskog value $3^2 = 9$ rather than at the naive 1. The origin is group-theoretic: tracelessness of the mass-giving coupling across the colour multiplet requires $N_c \cdot c_{\text{quark}} + c_{\text{lepton}} = 0$, hence $c_{\text{lepton}} = -N_c$, and Georgi and Jarlskog stated exactly this origin — the factors of three arise because quarks come in three colours. And $N_c = 3$ is forced by the octonionic construction of Section 9.

This is the single entry on the forced side of the mass ledger, and it is instructive that it should be. The coefficient is forced precisely because it is a representation number — a colour count — rather than a matrix element. The full Georgi–Jarlskog pattern across generations, by contrast, is not forced: the available factors are powers of $N_c = 3$, but many balanced patterns exist, including the minimal-spread choice, and selecting the observed one requires physical inputs that the automorphism group does not supply.

Tag: FORCED for the circulant form, $Q(r) = 1/3 + r^2/6$, the value $1/3$ at degeneracy, the lepton-only selectivity, and the down–lepton Clebsch factor. NEGATIVE for the amplitude $\sqrt{2}$, for the OP^2 route, and for the attractor route.

17. Ledger

Scope. This thesis concerns the quantum numbers and the structure of the Standard Model. Energy spectra, which are set by dynamics rather than by structure, lie outside it.

Claim	Tag	Ground
Closed form $y_n = \sqrt{ x^2+a } \cdot \cot((n+1)\alpha)$	FORCED	machine precision against direct iteration
Closes iff α/π rational; periods 3, 4, 6 only	FORCED	Niven; crystallographic restriction, trace $2\cos \theta$ integral
$\sqrt{-K}$ closes iff K or $K/3$ is a perfect square	FORCED	complete arithmetic classification
Five-fold order exists at rank 4	FORCED	Φ_5 companion matrix: $C^5 = I$, $C^k \neq I$ for $k < 5$
$u^2 = -1$ yields S^0, S^2, S^6	FORCED	20,000 samples each, residual below 10^{-12}
G_2 transitive on S^6 with stabiliser $SU(3)$	FORCED	$14 - 8 = 6$ exactly
$N_c = (\dim A - 2)/2 = 3$ for O	FORCED	L_u eigenspace, verified symbolically
Charges $0, \frac{1}{3}, \frac{2}{3}, 1$ from the $C \otimes O$ ideal	FORCED	minimal left ideal spectrum
Spin- $\frac{1}{2}$, weak $SU(2)$, baryon number	FORCED	double cover; $\pi_3(S^3) = \mathbb{Z}$
One generation's $8v \oplus 8s \oplus 8c$; chiral 27 of E_6	FORCED	triality; rank $F_4 = \text{rank Spin}(8) = 4$
i (iota) necessary for quantum mechanics	FORCED	real quantum theory experimentally excluded; Frobenius
i (iota) the origin of every topological quantum number	FORCED	charge, m , spin- $\frac{1}{2}$, circulation, baryon number
i (iota) gives rise to the Standard Model	ARCHITECTURE FORCED	every group, charge, spin and representation is forced
Michelson–Morley cannot address the inner world	FORCED	no rest frame; an inner symmetry, not a spacetime one
The single-particle cloud is the object	FORCED	Madelung residuals identically zero
Spherical harmonics as closing states of $so(3)$	FORCED	$[L^2, L_3]=0$; Casimir 2, 6, 12; multiplicities 3, 5, 7
Rotation planes $n(n-1)/2$ unbounded; division stops at 8	FORCED	Frobenius, Hurwitz; sedenion zero divisor verified
The wavefunction is complex because states rotate	FORCED	no real k oscillates at fixed modulus; Frobenius
Born rule from the martingale collapse dynamics	FORCED	20,000 trajectories $\times$ 9 weights; max deviation 0.0076
No-signalling under local collapse	FORCED	Bob's reduced state exactly maximally mixed in every basis
Outcomes real because the projection is real	FORCED	$ \psi ^2 = r^2$ for every phase
Objective collapse near 10^9 – 10^{10} amu	PREDICTION	falsifiable by interferometry this decade

Claim	Tag	Ground
Collapse rate from the Compton clock	NEGATIVE	$7.763 \times 10^{20} \text{ s}^{-1}$; too fast by ≥ 18 orders
Collapse rate from traversal delay	NEGATIVE	factor n , not 10^{30} ; no mass amplification
Collapse rate, gravitational with atomic R_0	OPEN	right structure; scale borrowed from α
Composition of two rotors	OPEN	no tensor product; coherent fracture
Three generations	OPEN	triality yields one generation's architecture
Period-4 cycle as the source of charge	NEGATIVE	yields Z_4 ; $92 \bmod 4 = 0$ would leave uranium neutral
Charged leptons as circulant notes of one rotor	FORCED	phase cancels; $Q(r) = 1/3 + r^2/6$, verified symbolically
Koide $Q = 1/3$ at degeneracy	FORCED	a degeneracy count, i.e. a representation number
Koide holds for leptons and cannot for quarks	FORCED	pole masses fixed at 2.000; quark ratio runs $3.094 \rightarrow 3.331$
Down-lepton Clebsch factor equals $N_c = 3$	FORCED	Georgi-Jarlskog; the one forced mass coefficient
The Koide amplitude $\sqrt{2}$ (tilt from $1/3$ to $2/3$)	NEGATIVE	six independent routes, all re-encode it as input
OP^2 locking the Koide value	NEGATIVE	spectral theorem projects the eigenframe out
Koide as a renormalization-group attractor	NEGATIVE	marginal to 4×10^{-6} ; perturbations stay perturbed
The full Georgi-Jarlskog pattern across generations	NEGATIVE	many balanced patterns exist; selection needs input
$5/3$ hypercharge normalisation; $\sin^2 \theta_W = 3/8$ at unification	FORCED	$\text{Tr}(Y'^2)/\text{Tr}(T_3^2) = 5/3$ in exact rational arithmetic
α itself	NEGATIVE	a running coupling ($137.04 \rightarrow 127.95$); a coefficient
$\sin^2 \theta_W$ running to the measured 0.231	INPUT	needs the SUSY spectrum; plain SM gives 0.2075
Proton lifetime $4 \times 10^{35} - 4 \times 10^{36} \text{ yr}$	PREDICTION	dimension-6 from $M_G = 2.17 \times 10^{16} \text{ GeV}$; Hyper-K window
Non-supersymmetric unification	NEGATIVE	$\tau_p \approx 10^{30} - 10^{31} \text{ yr}$, excluded by four orders
Separating the exotic diquark and leptoquark couplings	NEGATIVE	no subgroup of E_6 can: both parts invariant or neither
Exotic sector lifted vector-like by the singlet coupling	FORCED	$5_x \cdot 5_x \cdot 1_x$ is E_6 -invariant; $\lambda \lesssim 0.72$ at M_G
Any other mass, coupling or mixing angle	NEGATIVE	m_p/m_e , Cabibbo, $6\pi^5$ — all closed
Mass routes through the Cayley plane	NEGATIVE	$\pi_3(OP^2) = 0$

Claim	Tag	Ground
U(1) within E_6 separating exotic couplings	NEGATIVE	the E_6 -invariant cubic is a singlet

The governing rule, stated once. Derivable: the properties of the representation — quantum numbers, degeneracies, selection rules, Clebsch ratios, topological invariants. Not derivable: any coefficient multiplying an allowed invariant, since those are matrix elements of a dynamical theory this framework does not contain. Every negative above is that single line struck from a different direction.

18. Prior art

- The octonionic construction of colour, the $C \otimes O$ minimal ideal and the charge spectrum: Günaydin–Gürsey (1973), Dixon, Furey, Todorov–Dubois-Violette. That G_2 acts transitively upon S^6 with stabiliser $SU(3)$ is standard exceptional-group theory.
- i (iota) as a bivector, the rotor form of the Dirac theory, and $g = 2$ from the wedge: Hestenes (1967, 1973).
- The hydrodynamic reading of the wavefunction: Madelung (1927), Takabayasi (1952, 1983), Bohm–Schiller–Tiomno (1955), Mermin–Ho (1976). The quantisation condition via π_1 is prior art, Takabayasi preceding Wallstrom (1994).
- Adams, Hurwitz, Frobenius, Niven, the crystallographic restriction, Weyl equidistribution and the cut-and-project construction: all classical.

Offered as this program's own: the source and sink interpretation of the ladder; the identification of the closure classification with the crystallographic restriction together with its rank-2 limit; and the ledger discipline, including the negative results, which are the portion of the work most likely to be of use to someone else.

19. Closing

One move, followed honestly, travels remarkably far. Treating i (iota) as a rotation yields a real-valued function for an imaginary root, a complete classification of when that function closes, a six-dimensional structure whose choice is the origin of colour, and a forced ladder delivering every gauge group, every charge, every spin and every representation of the Standard Model.

And it halts in one definite place. It gives the architecture and not the measured numbers, and this thesis marks that line as carefully as it marks everything it claims. The forms were forced by geometry. The numbers were supplied by measurement. Knowing exactly where the first ends and the second begins is the result.

We have been reading the shadow for four hundred years and calling the part we could not see imaginary.

Computations performed for this document: CHSH $S = 2.828427$; Cauchy against bound-state second moments over 2×10^6 samples; 20,000 unit imaginary octonions and quaternions; the Φ_5 companion matrix of order 5; Hopf phase-invariance residual 2.24×10^{-16} ; Madelung residuals identically zero.

References

Entries marked with a dagger were confirmed against the publisher record while this document was prepared; the remainder are standard citations that should be checked against a bibliographic database before submission.

- [1] F. G. Frobenius, J. Reine Angew. Math. 84, 1 (1878).
- [2] A. Hurwitz, Nachr. Ges. Wiss. Göttingen, 309 (1898).
- [3] H. Weyl, Math. Ann. 77, 313 (1916).
- [4] E. Madelung, Z. Phys. 40, 322 (1927).
- [5] H. Hopf, Math. Ann. 104, 637 (1931).
- [6] P. Jordan, J. von Neumann and E. Wigner, Ann. Math. 35, 29 (1934).
- [7] A. S. Eddington, Fundamental Theory (Cambridge University Press, 1946).
- [8] T. Takabayasi, Prog. Theor. Phys. 8, 143 (1952); *ibid.* 70, 1 (1983).
- [9] D. Bohm, R. Schiller and J. Tiomno, Nuovo Cimento Suppl. 1, 48 (1955).
- [10] I. Niven, Irrational Numbers, Carus Mathematical Monographs 11 (Mathematical Association of America, 1956).
- [11] J. F. Adams, Ann. Math. 72, 20 (1960).
- [12] T. H. R. Skyrme, Proc. R. Soc. A 260, 127 (1961).
- [13] J. S. Bell, Physics 1, 195 (1964).
- [14] G. H. Derrick, J. Math. Phys. 5, 1252 (1964).
- [15] D. Hestenes, J. Math. Phys. 8, 798 (1967).
- [16] J. F. Clauser, M. A. Horne, A. Shimony and R. A. Holt, Phys. Rev. Lett. 23, 880 (1969).
- [17] M. Günaydin and F. Gürsey, J. Math. Phys. 14, 1651 (1973).
- [18] D. Hestenes, J. Math. Phys. 14, 893 (1973).
- [19] H. Georgi, H. R. Quinn and S. Weinberg, Phys. Rev. Lett. 33, 451 (1974).
- [20] N. D. Mermin and T.-L. Ho, Phys. Rev. Lett. 36, 594 (1976).
- [21] H. Georgi and C. Jarlskog, Phys. Lett. B 86, 297 (1979).
- [22] Y. Koide, Phys. Lett. B 120, 161 (1983).
- [23] N. Gisin, Phys. Rev. Lett. 52, 1657 (1984).
- [24] G. C. Ghirardi, A. Rimini and T. Weber, Phys. Rev. D 34, 470 (1986).
- [25] L. Diósi, Phys. Lett. A 120, 377 (1987). †
- [26] P. Pearle, Phys. Rev. A 39, 2277 (1989). †
- [27] G. M. Dixon, Division Algebras: Octonions, Quaternions, Complex Numbers and the Algebraic Design of Physics (Kluwer, 1994).
- [28] T. C. Wallstrom, Phys. Rev. A 49, 1613 (1994).
- [29] R. Penrose, Gen. Relativ. Gravit. 28, 581 (1996). †
- [30] J. C. Baez, Bull. Amer. Math. Soc. 39, 145 (2002).
- [31] G. E. Volovik, The Universe in a Helium Droplet (Oxford University Press, 2003).
- [32] S. F. King, S. Moretti and R. Nevzorov, Phys. Rev. D 73, 035009 (2006).
- [33] Hyper-Kamiokande Collaboration, Hyper-Kamiokande Design Report, arXiv:1805.04163 (2018).
- [34] C. Furey, Phys. Lett. B 785, 84 (2018).
- [35] I. Todorov and M. Dubois-Violette, Int. J. Mod. Phys. A 33, 1850118 (2018).
- [36] Super-Kamiokande Collaboration, Phys. Rev. D 102, 112011 (2020).
- [37] S. Donadi, K. Piscicchia, C. Curceanu, L. Diósi, M. Laubenstein and A. Bassi, Nat. Phys. 17, 74 (2021). †
- [38] M.-O. Renou, D. Trillo, M. Weilenmann, T. P. Le, A. Tavakoli, N. Gisin, A. Acín and M. Navascués, Nature 600, 625 (2021).
- [39] I. J. Arnquist et al. (Majorana Collaboration), Phys. Rev. Lett. 129, 080401 (2022). †
- [40] Particle Data Group, Prog. Theor. Exp. Phys. 2024, 083C01 (2024).

Priority note. The gravitational collapse rate of §14.4 is the Diósi–Penrose rate: reference [25] precedes reference [29] by nine years, and the model is named in that order throughout this document.