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From Geometry to Clocks to Scales

Rotor Circulation, the Binary Tetrahedral Group, and the Fermion Mass Spectrum

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Unified manuscript v1.0, 31 August 2026. Supersedes Geometry_to_Clocks_to_Scales (V8.12) and The Lepton Angle as the Unique Exportable Angle of E_6 (v0.8), which are combined here with a new central chain (Part II).

Abstract

We define mass as the modulus-squared of a complex rotor — a quantised circulation in a source/sink medium — and show that this definition, together with one axiom, that the imaginary unit acts as an integral rotation of the plane, organises the fermion mass spectrum in a way that is derived where it is derived and labelled as input where it is not. The Koide constant emerges as a scale-invariant angle, so its constancy is geometry and its value a single condition; each of the four fermion sectors is generated by one clock, one phase and one amplitude. The charged-lepton hierarchy angle $\delta = 0.2222212(20)$, consistent with $2/9$ radians at 0.5σ — an observation due to Brannen and Rosen — and with no rational fraction of a turn, is reached in this work by two roads: a phase law $\theta = u(1-u)$ with $1/u = N_c = 3$, and the ratio of E_6 's Weyl invariant degrees 2 and 9. The central result of this paper is that the two roads are one. The crystallographic restriction selects the binary tetrahedral group $2T$ as the largest finite subgroup of $SU(2)$ with only crystallographic element orders; McKay's correspondence turns $2T$ into the affine E_6 diagram, on which triality is tensoring with a character of $Ab(2T) = Z_3$ and N_c is the branch-node mark; deleting the branch node gives the trinification $SU(3)^3$, and E_6 is the unique simple Lie algebra admitting $SU(N)^N$ at maximal rank. The product of the three $SU(3)$ Coxeter elements is the ζ_3 -regular element of $W(E_6)$, whose centraliser is the Hessian group $G_{25} = 3^{1+2} \rtimes 2T$. The 27 weights of E_6 form a torsor for the Heisenberg group 3^{1+2} and project three-to-one onto the nine inflection points of the Hesse pencil, with quarks, antiquarks and leptons of one generation landing as one of the four Hesse triangles; the phase law's N_c^2 and the degree 9 of G_{25} are the order of the Heisenberg quotient. Every step is verified by deposited scripts. A pre-registered blind protocol over the canonical integer emissions of $\mathbb{C} \otimes \mathbb{O}$ finds $2/9$ as the unique hit among eighteen ratios; a no-go theorem excludes vacuum-alignment origins; a radian theorem licenses reading a dimensionless degree ratio as an angle. Sector clocks fix the scales as inputs, one unification clock at 2×10^{16} GeV threading the four sectors. The neutrino sector predicts normal ordering, $\Sigma m_\nu \approx 58$ meV, and a scale-free ratio $\Delta m_{31}^2 / \Delta m_{21}^2 = 31.88$ against 33.56 ± 0.64 measured, 2.6σ low after JUNO's first release; JUNO's forthcoming atmospheric determination decides it. We state what is not derived: $\chi = \pi/4$, all mass scales, three generations, the breaking of E_6 , the gauge couplings, and the quark angles.

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Addendum to the Abstract (v1.6). The status of the sector phases is upgraded in this version. The quantum-number law of Proposition 3, $1/u = N_c^{(3/2+T_3)} \cdot C_2(3)^q \rightarrow \{3, 4, 9, 12\}$, is now confirmed in every charged sector when each mass is evaluated at its own scale in the scheme of the interactions it feels — pole for leptons, $\overline{MS}$ at $\mu = m$ for quarks, floored at the condensate boundary ≈ 2 GeV: the measured $1/u$ are 2.9998(2), 4.004(57) and 11.93(12) against 3, 4 and 12, the up value landing at 0.58σ on the number this manuscript pre-registered. The coloured-sector phases are therefore no longer 'not established'. The single surviving deviation in the entire shape sector is the neutrino, $1/u = 8.868(47)$ against 9 — the -5% of the falsifier list — of which 27 % is now supplied by a computed one-loop seesaw-threshold running; JUNO decides the rest. A consistency fit of the surviving common-scale conditions gives $\mu^ = 3.3 \times 10^6$ GeV at $\chi^2 = 0.12/2$, with the dimension entering only through Λ_{QCD} by ordinary transmutation. Sections 19a–19d, the additions to Sections 24–26, 29 and 31, and Appendix D carry the details, the new predictions ($m_\tau = 1776.97$ MeV; $m_u = 2.75$ MeV own-scale; $\Delta m_{21}^2 = 7.88 \times 10^{-5}$ eV² if the heavy phase is exact), and the audit trail, including four retracted intermediate claims.*

Addendum (v1.7, 2 September 2026). The law is confirmed exactly in the odd-isospin sectors (0.4σ , 0.07σ) and strained by a common one per cent in both even-isospin sectors (up 2.2σ with FLAG/PDG inputs, neutrinos 2.8σ) — precisely the sectors carrying the factor of N_c that the binary tetrahedral group provably cannot supply, since 3, 4 and 12 are rotation orders of $2T$ and 9 is not. Inverted, the law is a dictionary returning the electric charges from the mass ratios to 10^{-4} ; on the Clifford occupation number its colour flag and isospin are the triplet condition and the grading. Tri-bimaximal mixing follows from the group's Z_3/Z_2 structure and is provably independent of the neutrino phase residual. The five sector clocks close into a two-branch seesaw through the two condensates' decay constants — exact on the electroweak branch, a 0.46σ candidate on the QCD branch predicting $m_d = 4.75$ MeV — leaving the spectrum's dimensionful content at two condensates, one unification scale and the top Yukawa. Details, proofs, eighteen scripts and the retraction record: companion thesis v9.

Relation to prior work

Three things in this paper are not ours and the reader should have them in view from the start. The circulant parametrisation of three masses and the observation that the charged-lepton phase is $2/9$ are Brannen's [Brannen 2006], independently noted by Rosen [Rosen 2007]; Żenczykowski [Żenczykowski 2012, 2013, 2018] developed the parametrisation for the quark sectors and proposed the phases $4/27$ and $2/27$ for down and up, with a lepton–quark sum rule — predictions this paper competes with and compares against in §20. The Koide relation as an angle is Foot's [Foot 1994]; Koide's own $U(3)$ -family and Yukawaon programmes [Koide 1990, 2006] are the standard dynamical approach. The binary tetrahedral group has a long history in particle physics — first gauged as a finite subgroup of $SU(2)$ by Case, Karplus and Yang [1956], and used as a flavour symmetry acting on the generation index since Frampton and Kephart [1995], with a large model-building literature [Eby, Frampton, Matsuzaki 2009; Natale 2016]. In this paper $2T$ is not a family symmetry: it

is the unit group of the integral quaternions, its McKay graph is the gauge algebra, and the generation Z_3 is a separate object (§28). The mathematics of Parts II is classical end to end and is cited where used: McKay [1980], Kronheimer [1989], Borel–de Siebenthal [1949], Springer [1974], Shephard–Todd [1954], and for the Hessian group and the Hesse pencil the survey of Artebani and Dolgachev [2009], which contains the structure $G_{25} = 3^{1+2} \rtimes \text{SL}(2, F_3)$ and the identification of the four triangles with the four order-3 subgroups that §9 and §10 recompute; the Heisenberg group of order 27 acting on $\mathbb{C}^3$ with the Hesse pencil and its relation to cubic (del Pezzo) surfaces, hence E_6 , is in the orbifold literature [arXiv:1001.2893; Douglas–Moore 1996]. Two contemporary programmes reach the Koide relation through structures this paper also uses and must be distinguished from it: Singh derives mass ratios from the characteristic equation of the exceptional Jordan algebra $J_3(\mathbb{O})$ [Singh 2020, 2021], the same object through which $\mathbb{C} \otimes \mathbb{O}$ gives E_6 here; and Makaryev [2026] derives $Q = 2/3$ from the Clifford torus in S^3 with the amplitude $\sqrt{2}$ fixed by an equal-area bipartition, close to the S^3 and Hopf-fibration framing of the rotor papers, and claims a Wolfenstein parameter $\sin(2/9)$. What is ours, as far as we have been able to determine, is stated in §30.

Part I. The rotor and the Koide angle

This Part establishes what is known about the Koide relation, shows that its content is an angle rather than a mass ratio, spreads it across the four sectors, and gives the rotor definition of mass on which the rest of the paper rests.

1. The Koide relation and what is established

For the charged leptons [1],

$$Q = \frac{m_e + m_\mu + m_\tau}{(\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau})^2} = 0.666661, \quad (1)$$

a deviation of -0.001% from $2/3$. The relation is reproduced by a circulant parametrization of the square-root masses [2,3,4],

$$\sqrt{m_k} = a \left(1 + \frac{b}{a} \cos \left(\theta + \frac{2\pi k}{3} \right) \right), \quad k = 0, 1, 2, \quad (2)$$

with $b/a = \sqrt{2}$ giving $Q = 2/3$; both the scale a and the phase θ cancel from Q . This parametrization, its extension to neutrinos with a sign choice [3], the sector-dependent behaviour across quark families [4], the connection of the sector pattern to the universal seesaw [5], and the inverse form for the down quarks [7] are all established results. We do not claim them. What we add is a geometric reading of the constancy of Q , a single physical picture for the sector split, and a definition of mass in terms of a rotor; these are the subject of the paper.

2. Q is a scale-invariant angle

Write $z = (\sqrt{m_1}, \sqrt{m_2}, \sqrt{m_3})$ and let $\mathbf{1} = (1,1,1)$. Then $Q = |z|^2/(z \cdot \mathbf{1})^2$, and since $z \cdot \mathbf{1} = |z|\sqrt{3}\cos\phi$ with ϕ the angle between z and $\mathbf{1}$,

$$Q = \frac{1}{3\cos^2\phi}. \quad (3)$$

Thus Q depends only on the *direction* of the square-root-mass vector, not on its length: the mass scale cancels. This is why Q is a stable, bounded ratio for any three positive masses—it is an angle, and angles are scale-free. This relation forces $Q \in [\frac{1}{3}, 1]$, with $Q = \frac{1}{3}$ at the democratic point $\phi = 0$ (equal masses) and $Q \rightarrow 1$ at the axial limit (one mass dominant). The charged leptons sit at $\phi = 45^\circ$, i.e. the equipartition point where the democratic and oscillating parts of the circulant form carry equal weight, $b^2/2 = a^2$. The *constancy* of Q is therefore forced; its *value* $2/3$ is the single condition $\phi = 45^\circ$, equivalently $b/a = \sqrt{2}$, which no symmetry within the circulant framework fixes (the decomposition that defines the ratio b/a and any symmetry that would fix it are mutually exclusive; this is proved as Proposition 6). The value is an input; the constancy is geometry.

3. Spreading the relation across sectors

The charged-lepton relation uses the charges $\sqrt{m}$; the down-quark [7] and neutrino relations are carried by the *inverse* charges $1/\sqrt{m}$. In one line, complex inversion explains the split. On the unit circle $\bar{z} = z^{-1}$: charge conjugation and inversion coincide. The neutral and down-type sectors are exactly those for which the neutral-sector structure makes conjugation and radial inversion available—neutrinos admit Majorana mass ($\nu = \nu^c$) and acquire it inversely through the seesaw $m_\nu \sim v^2/M$ [5], both features forbidden to the charged (Dirac) leptons, which therefore keep the direct form. Algebraically, the inverse relation on the light neutrinos equals the direct relation on the heavy Majorana masses, $Q^{-1}\{m_\nu\} = Q\{M_i\}$.

Measured in each sector's proper form, three sectors meet Koide and one does not:

Sector	Form	Q	b/a
Charged leptons	Direct	0.6667	$\sqrt{2}$
Down quarks	inverse	0.6652	$\sqrt{2}$
Neutrinos	inverse	0.6670	$\sqrt{2}$
Up quarks	direct	0.849	$\sim \sqrt{3}$

The up sector departs to $\phi \simeq 51^\circ$ ($Q \simeq 0.85$), near the axial limit. This is not a free failure: the up sector is the only one whose heaviest member has a Yukawa coupling of order unity ($y_t \simeq 0.99$; every other fermion has $y \ll 1$). Two consequences follow, both standard. The steep hierarchy $m_t/m_u \sim 10^5$ tilts the square-root-mass vector nearly onto one axis, pushing Q toward the singular limit rather than the moderate value $2/3$; and the $O(1)$ top

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Yukawa drives large non-universal running that pulls the up ratios apart, so no scale restores a clean value (we have checked the running across the full range up to the Planck scale). The other three sectors have uniformly small Yukawas, run near-universally, and preserve their ratios. The up sector's departure is thus the Standard Model's own singling-out of the top, not a defect of the relation.

4. A rotor definition of mass

We now give mass a definition within the rotor reading, in which the imaginary unit is the generator of a rotation and a particle is a self-sustaining circulation—a rotor. The setting is the Clifford-algebraic description of the Standard Model fermions: one generation is a minimal left ideal of the complex Clifford algebra $\mathbb{C}\ell(6) \cong M_8(\mathbb{C})$, represented via octonionic left-multiplication on the eight complex dimensions of the complexified octonions $\mathbb{C} \otimes \mathbb{O}$ — a representation, not an algebra isomorphism, since $\mathbb{C} \otimes \mathbb{O}$ is non-associative and is not itself a Clifford algebra; it serves as the spinor module on which the associative $\mathbb{C}\ell(6)$ acts, the octonionic multiplication organizing the minimal ideals into one generation (after Furey, Todorov, and Dixon) — acting on the Fock space of the octonionic chain, in which the unbroken gauge symmetries $SU(3)_c \times U(1)_{em}$ arise as the subgroup of $Spin(6)$ commuting with a chosen complex structure. A rotor is an element of this algebra restricted to the phase circle generated by a single imaginary unit i (with $i^2 = -1$), and the sector's three generations are the three $\mathbb{Z}_3$ -related modes of that circle. The mass definition below is thus a statement about the norm form of the Clifford algebra: the rotor's mass is the algebra's quadratic (modulus) form evaluated on the rotor element, and the colour degrees of freedom are the transverse imaginary units $e_2, \dots, e_7$ on which that quadratic form does not depend—the algebraic origin of the colour-blindness proven in Sec. 5. In this sense the circulant spectrum, the scale-invariant angle Q , and the mass definition are all properties of the norm form of $\mathbb{C} \otimes \mathbb{O}$; the hydrodynamic language of circulation renders them physically, but the underlying object is Clifford-algebraic. Let the rotor be a complex field $z = |z| e^{i\varphi}$. Its elastic (vibrational) energy is quadratic in the field; for a complex mode the real, positive quadratic is the modulus-squared $z\bar{z} = |z|^2$, not z^2 . Equating this energy to mc^2 ,

$$\boxed{m = \frac{k}{2c^2} z\bar{z} = \frac{k}{2c^2} |z|^2,} \quad (4)$$

where k is the rotor stiffness (the dimensionful scale of the sector). Three features make the mass definition the natural definition.

The phase cancels; mass is Lorentz invariant.

Because $z\bar{z} = |z|^2$ is independent of the rotor phase φ ($e^{i\varphi} e^{-i\varphi} = 1$), the mass does not depend on where the rotor points. This is the same statement as $m^2 = \det J$ for the rotor current J : mass is the boost-invariant magnitude of the circulation. The complex form makes this manifest rather than assumed—the invariance of mass *follows* from taking the modulus.

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Definition 1 (Mass). *The mass of a rotor is the stiffness times the modulus-squared of the complex rotor field, $m = (k/2c^2) z\bar{z}$; equivalently, the Lorentz-invariant magnitude of the rotor current, $m^2 = \det J$. It is the elastic energy of the trapped rotor vibration, divided by c^2 .*

The observable is the real projection.

For the three generations the rotor carries three modes, and the masses that reproduce the data are the *real* (Joukowski) projections of the rotor circle, $w = \frac{1}{2}(z + z^{-1}) = \cos\theta$, giving the circulant form: $m_k = (k/2c^2) a^2 [1 + \sqrt{2}\cos(\theta + 2\pi k/3)]^2$. The full complex phasor does not reproduce the spectrum; the observable mass is the rotor's real shadow. The lightest lepton sits near a node of this three-fold standing wave (its mode factor is 0.04), which is why it is light: not a small stiffness, but a near-cancellation of the circulant.

What the definition fixes and what it does not.

The definition the mass definition gives mass its physical content—the elastic energy of a trapped rotor—and, through the circulant and angle relations, the *form* of the spectrum (a circulant), the *constancy* of Q (an angle), and the *sector split* (conjugation and seesaw). It does not fix the stiffness k (the dimensionful scale, one per sector: $k_{\text{lept}}^{1/2} \sim \sqrt{2\kappa^2}$ with $\kappa^2 \simeq 314$ MeV for the charged leptons) or the phase θ (empirically $2/9$ for the leptons, exact and renormalization-stable, but not derived). These are inputs, exactly as the scale v and the Yukawa couplings are inputs in the Standard Model. The rotor gives mass a definition and the spectrum a form; it does not manufacture the two dimensionful/structural numbers that locate them.

Part II. From the integral rotor to E_6 and the nine inflection points

This Part is new. From the single axiom of §5 it computes a chain with no free choice at any step: the crystallographic restriction selects $2T$; McKay turns $2T$ into E_6 ; the trinification, the Hessian group, and the Hesse configuration follow; and the two prior derivations of the lepton angle are shown to be one.

5. The axiom: an integral rotor

The framework's starting point is that the rotor is integral. The earlier statement of this axiom had the imaginary unit act on a rank-2 lattice $\mathbb{Z}^2$, with Niven's theorem then giving the crystallographic periods 1, 2, 3, 4, 6. That statement is correct for each element of the groups that follow, but it cannot be the lattice the group preserves: no finite subgroup of $\text{GL}(2, \mathbb{Z})$ has order above 12, and a plane lattice admits a 90° rotation or a 60° rotation but never both, whereas $2T$ contains both. The rotor's integral structure is rank 4. The double cover lifts the rotor from $\mathbb{C}$ to $\mathbb{H}$, and the integral quaternions are the Hurwitz order — the quaternions whose four components are all integers or all half-integers. Its unit group is $2T$ (§6). Every unit of a $\mathbb{Z}$ -order is an algebraic integer of norm one, so its trace is a rational integer and its

order is crystallographic automatically: the restriction is a theorem about the Hurwitz order, not a postulate about the plane.

The two halves of the order are the two contributions the framework has always separated. The Lipschitz order $\mathbb{Z}^4$ — integer components only — has unit group Q_8 , of orders $\{1, 2, 4\}$. The sixteen half-integer units $(\pm 1 \pm i \pm j \pm k)/2$ are exactly the elements of order 3 and 6. The double cover's contribution is the half-integer lattice points. Five-fold order is absent because the icosian ring is a $\mathbb{Z}[\varphi]$ -order, rank 8 over $\mathbb{Z}$ — the E_8 lattice — and the maximal $\mathbb{Z}$ -order in the quaternions is unique. §32 gives the physical reason the medium selects the Hurwitz order.

6. The double cover and the binary tetrahedral group

The double cover $SU(2) \rightarrow SO(3)$ is the framework's one contribution that is not already implied by $SO(3)$ non-commutativity: it admits half-integer winding, binary polyhedral groups, and 4π periodicity. The finite subgroups of $SU(2)$ are the cyclic groups Z_n , the binary dihedral groups $2D_n$, and the binary polyhedral groups $2T$, $2O$, $2I$ of orders 24, 48, 120. Applying the axiom of §5 to the defining representation — every element's order must be crystallographic — gives the table below (computed from quaternion realisations).

group	element orders	crystallographic	McKay type
$Z_1 \dots Z_6$	$\subset \{1,2,3,4,6\}$	yes	A (cycle, no branch point)
$Q_8 = 2D_2$	$\{1,2,4\}$	yes	D_4 affine
$2D_3$	$\{1,2,3,4,6\}$	yes	D_5 affine
$2T$	$\{1,2,3,4,6\}$	yes	E_6 affine
$2O$	$\{1,2,3,4,6,8\}$	no	E_7 affine
$2I$	$\{1,2,3,4,5,6,10\}$	no	E_8 affine

$2T$ is the largest finite subgroup of $SU(2)$ all of whose elements have crystallographic order — order 24 against 12 for $2D_3$. Scaled by $\sqrt{2}$ its 24 elements are the D_4 root system, the 24-cell: $2T$ is at once the largest crystallographic subgroup of $SU(2)$, the unit group of the maximal integral quaternion order, and the minimal-vector shell of D_4 . It is also isomorphic to $SL(2, F_3)$, a fact used in §6. This is the only selection in the paper, and it is the framework's own theorem rather than an imported condition: $2O$ needs the octagonal period, $2I$ the pentagonal, and neither is available at rank 2.

On E_8 . The same restriction excludes $2I$ and therefore, through McKay, E_8 — at rank 2. Since five-fold order is crystallographic at rank 4, E_8 is available one rank up, where the earlier framework paper uses $E_6 \subset E_8$ for the generation structure. E_6 is the rank-2 story told here; E_8 is a containing structure at higher rank. The two are not in conflict provided each group is assigned to the rank at which it is crystallographic.

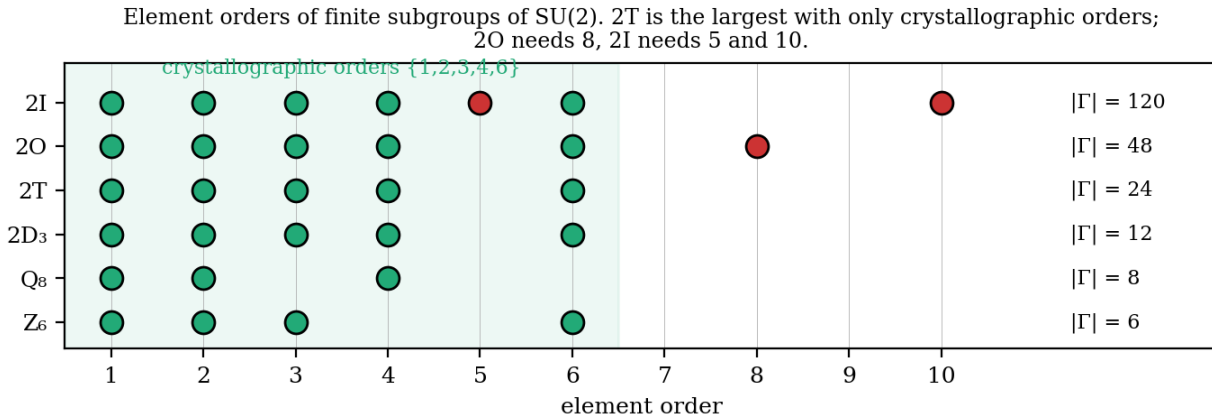


Figure 1. Element orders of the finite subgroups of $SU(2)$. Green: crystallographic orders $\{1,2,3,4,6\}$; red: orders unavailable at rank 2. $2T$ is the largest subgroup lying entirely in the green band. Computed from quaternion realisations (*why_2T.py*, *mckay_ade.py*).

7. McKay: $2T$ becomes the affine E_6 diagram

The McKay graph of a finite $\Gamma \subset SU(2)$ has the irreducible representations of Γ as nodes, joined by the multiplicity of ρ_j in $\rho_i \otimes 2$ where 2 is the defining representation. Computing characters by the Burnside–Dixon class-matrix method and forming the adjacency matrix gives, for $2T$, seven nodes of dimensions $(1,1,1,2,2,2,3)$, a symmetric adjacency with Perron eigenvalue exactly 2 and $A \cdot d = 2d$ — the affine E_6 diagram, with the irreducible dimensions as the Dynkin marks ($\Sigma d = 12 = h$, $\Sigma d^2 = 24 = |2T|$). For $2O$ and $2I$ the same computation gives affine E_7 and E_8 . Ordinary $T, O, I \subset SO(3)$ have no two-dimensional representation and give no ADE diagram: the double cover is load-bearing here.

Both threes of this paper are visible on the $2T$ graph. Tensoring the irreducibles with a non-trivial character of $\text{Ab}(2T) = Z_3$ permutes them with cycle type $[3,3,1]$ — the three one-dimensional representations cycle, the three two-dimensional ones cycle, the three-dimensional one is fixed. That rotation of the three arms is triality, of order $|\text{Ab}(2T)| = 3$. Each arm is a chain of two nodes, an A_2 diagram, so each factor is $SU(3)$ with Coxeter number 3; the branch node carries mark 3, the dimension of $2T$'s largest irreducible. The number of arms, the Coxeter number of an arm, and the branch mark are all 3, and they are equal as a property of $2T$: for $2O$ the arms have lengths $(1,3,3)$ and the branch mark is 4; for $2I$, $(1,2,5)$ and 6. Among the binary polyhedral groups the symmetric star belongs to $2T$ alone.

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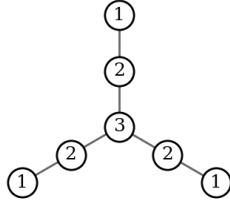
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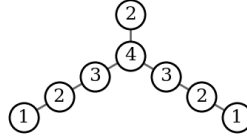
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McKay graphs of the binary polyhedral groups; node labels are irreducible dimensions (= Dynkin marks).
Only 2T is a symmetric star: #arms = arm-Coxeter number = branch mark = 3.

2T $\rightarrow$ affine E_6
arms [2, 2, 2], branch mark 3, $|Ab| = 3$



2O $\rightarrow$ affine E_7
arms [1, 3, 3], branch mark 4, $|Ab| = 2$



2I $\rightarrow$ affine E_8
arms [1, 2, 5], branch mark 6, $|Ab| = 1$

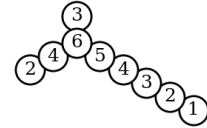


Figure 2. McKay graphs of 2T, 2O and 2I computed from their character tables; node labels are irreducible dimensions, which coincide with the affine Dynkin marks. Only 2T is a symmetric star, with number of arms, arm Coxeter number and branch mark all equal to 3 (*mckay_ade.py*, *tau_residual.py*).

8. Trinification and the uniqueness of E_6

Deleting the branch node from the affine E_6 diagram (Borel–de Siebenthal) leaves three A_2 diagrams: the maximal-rank subgroup $SU(3)_C \times SU(3)_L \times SU(3)_R$. In the Bourbaki labelling with highest root $\theta = \alpha_1 + 2\alpha_2 + 2\alpha_3 + 3\alpha_4 + 2\alpha_5 + \alpha_6$ and affine node $\alpha_0 = -\theta$ attached to α_2 , the three factors have simple roots $\{\alpha_1, \alpha_3\}$, $\{\alpha_5, \alpha_6\}$, $\{\alpha_2, \alpha_0\}$; they are mutually orthogonal six-element A_2 subsystems partitioning eighteen of the 72 roots.

Scanning every affine ADE diagram up to rank 20 and every node deletion for a remainder consisting of k copies of A_{N-1} with $k = N$ returns exactly one case: E_6 , $N = 3$. (Among non-simply-laced algebras B_2 and G_2 admit $SU(2)^2$, $N = 2$; no algebra admits $SU(N)^N$ for $N \geq 4$.) So ‘the algebra is E_6 ’ and ‘the number of $SU(N_c)$ factors equals N_c ’ are one condition, not two. The equality of the two threes cannot be tested inside E_6 because there is nothing else there; it is settled at 2T in §4.

9. The regular element and the Hessian group

Springer’s theory: an element $w \in W(E_6)$ is ζ_3 -regular if it has a ζ_3 -eigenvector lying in no reflection hyperplane. Since 3 divides exactly three of the degrees (2,5,6,8,9,12) of E_6 — namely 6, 9, 12 — the ζ_3 -eigenspace of a regular element is three-dimensional, and its centraliser is the complex reflection group with those degrees: Shephard–Todd G_{25} , the Hessian group, of order $6 \cdot 9 \cdot 12 = 648$.

Generating $W(E_6)$ (51840 elements) and classifying its 800 elements of order 3 by ζ_3 -eigenspace dimension gives classes of sizes 240, 480, 80 (dimensions 1, 2, 3), with

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centralisers 216, 108, 648. The 72 order-3 elements that cyclically permute the three trinitification factors all lie in the 480-class: eigenspace dimension 2, not regular, centraliser 108. So trinitification triality is not the regular element, and no change of basis makes it so. But every one of the 72 commutes with some regular element, hence lies inside some G_{25} ; together they generate $Z_3 \times Z_3$; and the triality's two-dimensional eigenplane is a hyperplane inside the three-dimensional Hessian plane.

The identification that closes the gap is this. Each $SU(3)$ factor has a Coxeter element c of order 3; the product $c_C \cdot c_L \cdot c_R$ has eigenvalues ζ_3 and $\bar{\zeta}_3$ three times each, its ζ_3 -eigenvector has minimum root pairing 0.538, it lies in the 80-class, and its centraliser has order 648. The centre of the Hessian group is the diagonal $SU(3)$ Coxeter element. Further, $\langle c_C, c_L, c_R \rangle \rtimes \langle \tau \rangle$ is exactly the Sylow 3-subgroup of G_{25} (order 81, set equality), and $O_3(G_{25})$ — the intersection of the four Sylow 3-subgroups — is the extraspecial group 3^{1+2} of order 27, generated by τ and the traceless elements $c_C^a c_L^b c_R^c$ with $a+b+c \equiv 0 \pmod{3}$, with commutation relation $[\tau, c_C] = c_L c_C^{-1}$ and centre $\langle c_C c_L c_R \rangle$. Its quotient is the Hesse $Z_3 \times Z_3 = \langle [\tau] \rangle \times \langle [c_L c_C^{-1}] \rangle$. Finally G_{25}/O_3 has order 24 with element-order distribution $\{1:1, 2:1, 3:8, 4:6, 6:8\}$, identical to $2T = SL(2, F_3)$. So $G_{25} = 3^{1+2} \rtimes 2T$: the group that generates E_6 through McKay is the group that acts on the Hesse points modulo translations.

The self-closure is a consistency check, not a selection. $2I = SL(2, F_5)$ closes the same chain through E_8 , whose degrees divisible by 5 are $\{20, 30\}$, with centraliser $G_{16} = Z_5 \times 2I$ of order 600. What selects $2T$ over $2I$ is §3.

The 800 elements of order 3 in $W(E_6)$ by ζ_3 -eigenspace dimension.
Triality and the regular element are in different classes; every τ commutes with some regular element.

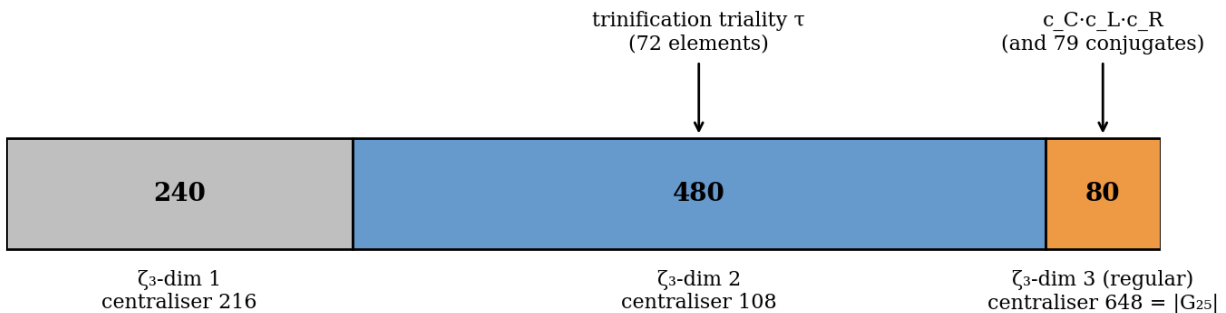


Figure 3. The 800 elements of order 3 in $W(E_6)$, by ζ_3 -eigenspace dimension. Trinification triality lies in the 480-class (centraliser 108); the product of the three $SU(3)$ Coxeter elements lies in the 80-class of regular elements (centraliser 648 = $|G_{25}|$). Every triality element commutes with some regular element (*trinitification_vs_regular.py*, *seal_coxeter.py*).

10. The 27 on the nine inflection points

The 27 weights of E_6 's fundamental representation are the W -orbit of ω_1 . Under trinification, $27 = (3,3,1) + (3,1,3) + (1,3,3)$ — in the standard assignment, the quarks, the antiquarks, and the leptons together with the Higgs doublets of one generation. Each nine-element piece is picked out as the set of weights vanishing under the third factor.

The Heisenberg group $O_3(G_{25})$ of order 27 acts on the 27 weights simply transitively: no non-identity element fixes any weight. The 27 is a torsor for 3^{1+2} . Projecting onto the Hessian plane — the ζ_3 -eigenspace of $c_C c_L c_R$, with projector $(1 + \zeta_3 w + \zeta_3 w^2)/3$ — is the quotient by the centre: the 27 weights land on exactly nine distinct projective points, three weights each, the fibres being the centre's orbits.

Those nine points are recognised coordinate-free. They have exactly 12 collinear triples; every pair lies on exactly one line; four lines pass through each point; and the 9×10 matrix of cubic monomials evaluated at them has rank 8, so they are the base locus of a pencil of cubics. This is the Hesse configuration, the nine inflection points of a plane cubic, the affine plane $AG(2,3)$. Among its twelve lines there are exactly four triples of mutually disjoint lines — the Hesse triangles — corresponding to the four order-3 subgroups of $Z_3 \times Z_3$. On the quark, antiquark and lepton weights c_R, c_L, c_C respectively act trivially, so on each multiplet the regular element reduces to a product of two Coxeter elements with a two-dimensional ζ_3 -eigenspace: each multiplet projects onto a line. The three lines are disjoint, cover all nine points, and form the triangle of the subgroup $\langle [c_L c_C^{-1}] \rangle$; triality's triangle is a different one.

Hesse pencil	E_6 and physics
nine inflection points	the 27 weights modulo the centre $\langle c_C c_L c_R \rangle$
fibre of three over each point	a centre orbit $\{(a,i), (a+1,i+1), (a+2,i+2)\}$
one triangle (three lines)	the trinification: quarks, antiquarks, leptons
the three points on each line	the three orbits of the centre $c_C c_L c_R$ on the multiplet's 3×3 index grid — the diagonal Z_3 charge, $(i - j) \bmod 3$ when both factors enter with the same conjugation type, $(i + j) \bmod 3$ when opposite
translation along a line	$c_L c_C^{-1}$, the colour-to-left relative Coxeter element
translation between lines	triality τ
the other three triangles	the other three order-3 subgroups of $Z_3 \times Z_3$

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Hesse pencil	E_6 and physics
translations modulo the plane	$2T = \mathrm{SL}(2, F_3)$, the level-3 modular group

The lepton line and the neutrino sector. Reading the nonet $(1,3,3)$ as the standard trinification matrix L_{ij} — rows the $SU(3)_L$ index with rows 1, 2 the $SU(2)_L$ doublet, columns the $SU(3)_R$ index, column 1 holding H_d and e^c , column 2 holding H_u and ν^c , column 3 holding (ν, e) and the singlet N — the three fibres over the three lepton points are, computed from the Dynkin labels of the nine weights:

lepton point	fibre
P0	ν, e^c, H_u^0
P1	ν^c, e, H_d^0
P2	N, H_d^-, H_u^+

The three neutral fermions of the doubly-levered neutrino sector — ν , its Dirac partner ν^c , and the singlet N — occupy one Hesse point each. The relative Coxeter element acts on the nonet as c_L (c_C is trivial there) and steps $\nu \rightarrow e \rightarrow N$ down the third column: the doublet's two members and then the singlet, threaded by one generator of the Hesse $Z_3 \times Z_3$. The naming of columns depends on how hypercharge is embedded in $SU(3)_R$ and is convention up to a permutation of the three points; the pattern — one neutral fermion per point, doublet and singlet on one line — is not.

The mathematics here is classical — Hesse's configuration and the 27-torsor for the Heisenberg group go back through the 27 lines on a cubic surface to Coble and Dickson. What is new is the dictionary, and that it is computed from the Cartan matrix rather than asserted.

The nine inflection points in torsor coordinates $(a,b) \in \mathbb{Z}_3 \times \mathbb{Z}_3$, computed from the 27 weights of E_6 .
Solid rows = the trinification triangle (one multiplet per line); grey = the other nine lines (dotted where they wrap).

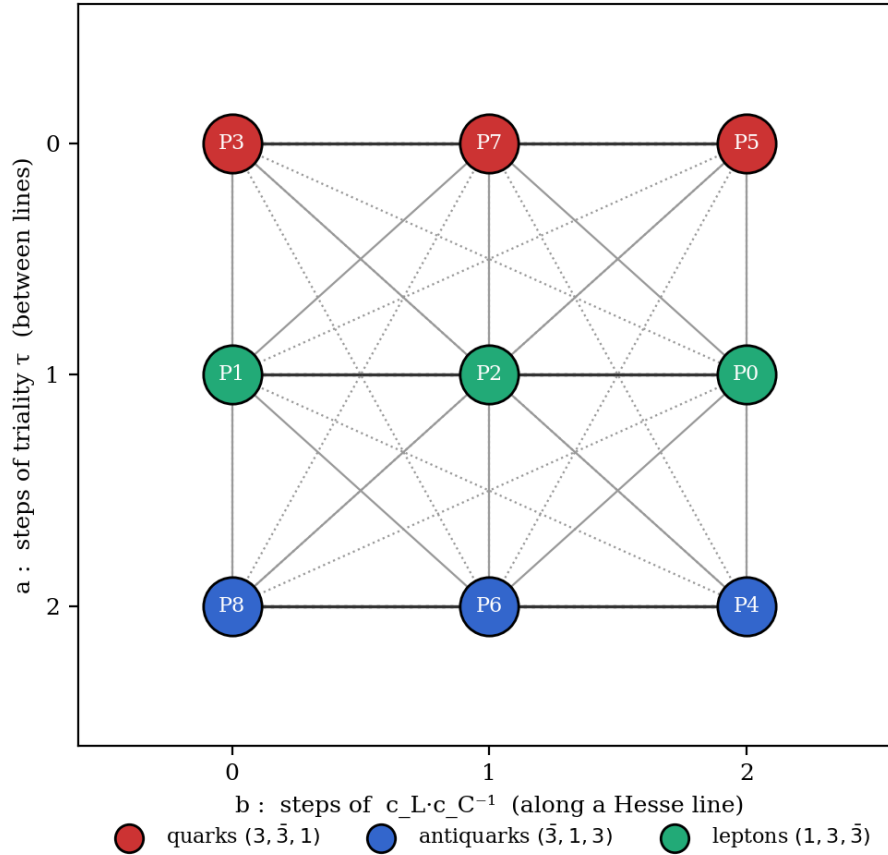


Figure 4. The nine inflection points in torsor coordinates $(a, b) \in \mathbb{Z}_3 \times \mathbb{Z}_3$, computed by projecting the 27 weights of E_6 onto the Hessian plane and acting with triality τ (rows) and the relative Coxeter element $c_L \cdot c_C^{-1}$ (columns). Point labels are the indices of `hesse_inflection_points.py`. The three solid rows are the trinification triangle — one multiplet per Hesse line; the other nine lines are grey, dotted where they wrap the torus. Every pair of points lies on exactly one of the twelve lines.

11. The two nines

The earlier framework paper derives the lepton angle from a phase law $\theta = u(1-u)$ with $1/u = N_c^{(1+T_3+\frac{1}{2}) \cdot C_2(3)q}$, which for the leptons gives $u = 1/3$ and $\theta = 2/9$; its 9 is N_c^2 . The prior E_6 paper derives $2/9$ as the ratio of the universal metric degree 2 to the unique conjugation-odd degree 9 surviving on the ζ_3 -regular plane; its 9 is a Shephard–Todd degree. A counterfactual in N_c appears to separate them — $(N_c-1)/N_c^2$ tracks N_c while $\deg_2/\deg_9$ is fixed — but that counterfactual is unavailable: there is no $\mathbb{Z}_4$ inside $\mathbb{Z}_3 \times \mathbb{Z}_3$, and by §8 there is no E_6 -like algebra for $N_c \neq 3$ at all.

What §9 and §10 establish is that both nines are $|O_3/Z(O_3)| = |Z_3 \times Z_3|$, the order of the Heisenberg quotient, realised as the nine inflection points. O_3 is the Heisenberg group over F_3 : order N_c^3 , centre of order N_c , quotient of order N_c^2 . The field is F_3 because every generator has order 3, and every generator has order 3 because the Coxeter number of $SU(N_c)$ is N_c . Along a Hesse line the step is the relative Coxeter element $c_L c_C^{-1}$ of order N_c ; between lines the step is triality, of order $|Ab(2T)| = 3$; and those coincide as a property of $2T$ (§7). The phase law's N_c^2 and the degree 9 of G_{25} are the same nine points with the same three lines through them.

The no-go theorem of the prior paper (§13) excludes vacuum-alignment mechanisms whose minima sit at symmetry-pinned rational turns. The phase law is not of that kind: it is a Green-dressed holonomy integral evaluated on representation data — exit (i) of the trichotomy, the only class the theorem leaves open. The two derivations are compatible, and the present paper shows they are one.

Part III. The lepton angle

This Part reproduces in full the derivation of 2/9 from E_6 : the measured angle, the no-go theorems closing dynamical origins, the pre-registered blind protocol, the selection theorem for the pair (2,9), the radian theorem, and the generation operator that transmits the cell to the spectrum.

12. The measured angle

Any three positive masses admit the exact parameterization

$$\sqrt{m_k} = (2E/\sqrt{3}) (\cos \chi + \sqrt{2} \sin \chi \cos(\delta + 2\pi k/3)), k = 0, 1, 2, (1)$$

in which E carries the scale and the pair (χ, δ) carries the shape: χ is the angle between the square-root-mass vector and the democratic direction $(1,1,1)$, and δ is the phase of the Z_3 charge-1 Fourier mode of the generation circulant. The parameterization is complete — three parameters, three masses — so (1) asserts nothing by itself; its value is that it separates the spectrum's shape into a radial datum and an angular datum whose behaviours, we will show, differ by theorem.

For the charged leptons ($m_e = 0.51099895$ MeV, $m_\mu = 105.6583755$ MeV, $m_\tau = 1776.93 \pm 0.09$ MeV, pole masses) the radial datum is the Koide relation [Koide 1981]: $\cos^2 \chi = (\Sigma \sqrt{m})^2 / (3 \Sigma m) = 0.500002$, i.e. $\chi = \pi/4$ to two parts in 10^6 — equivalently, the amplitude ratio of oscillating to constant part in (1) is $\sqrt{2}$ to five decimal places, the mass distribution is at exactly 50% fidelity with the uniform distribution, and the square-root-mass vector is null with respect to the Lorentzian form $(3/2)1 - J$ on flavour space, whose signature $(-,+,+)$ makes the democratic direction timelike [Foot 1994]. The angular datum is

$$\delta_1 = 0.2222212(20), (2)$$

the uncertainty dominated by m_τ . Both angles are invariant under flavour-universal running — (1) is homogeneous of degree zero in a common mass rescaling — so (2) is scheme-robust; E runs. We flag at the outset the known tension this exactness carries: flavour-non-universal QED self-energies should displace the pole-mass Koide relation at the $\sim 10^{-2}$ level, while it holds at $10^{-5.5}$; the standing proposal is a cancelling family-gauge sector [Sumino 2009]; quantitatively, $Q_l(m_Z) \approx 1.002 Q_l(\text{pole})$ [Smirnov 2006 review], while the quark-sector shape coordinates used below are protected by flavour-universal QCD running to the $\sim 0.2\text{--}0.3\%$ scheme bands quoted with them. Nothing below resolves this; we note it as independent evidence that the angular structure of (1) is protected by physics beyond the Standard Model.

The starting observation of this paper is arithmetic. The value (2) is consistent at 0.5σ with the rational number $2/9$ — of radians. It is not consistent with any rational fraction of a turn: the nearest rational multiple of π with denominator up to 100, $(7/99)\pi$, lies 44σ away, and the nearest rational value of $\cos 3\delta$ with denominator up to 50, $11/14$, lies 47σ away. An angle that is a ratio of two small integers in radian units, and provably not a fraction of the circle, is a peculiar object: every phase that group theory manufactures — holonomies, discrete-symmetry rotations, winding fractions — is a rational turn. Section 13 makes this peculiarity a theorem about origins; the remainder of the paper locates the two integers.

13. Where the angle cannot come from

We first close the dynamical routes. Let $H = \sum_a c_a \Lambda_a$ be an adjoint flavon of a flavour $SU(3)$, the minimal field content whose vacuum can set (χ, δ) [Koide 1990].

Theorem A (invariant ring). The ring of $SU(3)$ -invariant polynomials in H is generated by $I_2 = \text{Tr } H^2$ and $I_3 = \text{Tr } H^3$; in particular $\text{Tr } H^4 = I_2^2/2$ identically (Cayley–Hamilton for traceless 3×3), verified numerically to machine precision. All angular dependence of any invariant potential enters through the single combination $I_3 = (2|c|^3/\sqrt{3}) \cos 3\delta$.

Theorem B (renormalizable no-go).**** The renormalizable potential is $V = aI_2 + bI_2^2 + \kappa I_3$ — linear in I_3 , since $I_2 I_3$ has dimension five and I_3^2 dimension six. At fixed $|c|$, V is monotone in $\cos 3\delta$, so the minimum is pinned to the boundary: $\delta \in \{0, \pi/3\}$, a degenerate eigenvalue pair. No observed sector is degenerate and no observed angle is 0 or $\pi/3$: no renormalizable single-adjoint dynamics produces any flavour angle in nature. (This retro-explains the omission of the cubic in the classic potentials: including it renormalizably worsens the vacuum.)

Theorem C (relocation at dimension six). The first operator admitting an interior minimum is I_3^2/Λ^2 , giving $\cos 3\delta_{\min} = -k_3/2k_6$. Every observed angle becomes attainable — the charged leptons at $k_3/k_6 = -1.571775$, the up sector near -1.950 , the down sector near -1.89 to -1.91 (the quark figures carrying the scheme and frame ambiguities detailed in §21) — and none is forced: the angle relocates into a coupling ratio. (The proximity of -1.5718 to $-\pi/2$ is itself executed by the data: exact $-\pi/2$ maps to $\delta = 0.222486$, excluded at $\sim 130\sigma$.)

Theorem D (alignment and torsion). Enlarging to two adjoints H_1, H_2 enlarges the invariant ring by the mixed cubics; on the Cartan every cubic invariant is a single harmonic in the sum of angles, $\text{Tr}[h(t_1)h(t_2)h(t_3)] = (2/\sqrt{3}) \sin(t_1+t_2+t_3)$ (residual 8×10^{-16}). Full sixteen-dimensional minimization of the general renormalizable two-adjoint potential shows: (i) vacua commute ($||[H_1, H_2]|| \sim 10^{-8}$); (ii) interior angles exist but move continuously with couplings ($\delta: 0.69377 \rightarrow 0.69114 \rightarrow 0.68859$ under $\sim 2\%$ coupling steps) — permits, never forces; (iii) imposing exchange symmetry does not pin the angle, and coupling-independent extrema can lie only at fixed points of a residual symmetry acting on the flavour torus: rational multiples of π . Combined with the $44\sigma/47\sigma$ exclusions of §12, no alignment mechanism, discrete flavour symmetry, or holonomy can produce the measured angle.

The chain terminates in a trichotomy. A forced value of δ must be (i) quantized — a ratio of like-dimensioned integer data (weights, charges, degrees), the only class compatible with a rational-radian value; (ii) aligned — closed by Theorem D; or (iii) inherited from a parent algebraic structure whose canonical data include the integers. The remainder of the paper tests (i) and (iii) jointly, under a protocol designed to make the test's own trials countable.

14. The blind protocol and the hit

Exit (i) of the trichotomy invites a familiar failure mode: the space of small-integer constructions is dense enough that a sought number will be found, and finding it will mean nothing. (We quantified this backdrop before designing the protocol: unrestricted products and quotients of at most three natural group-theoretic atoms hit $2/9$ exactly in 76 distinct ways.) The test below was therefore run blind: the candidate integer data of the parent structure were enumerated and status-tagged with the targets masked, the comparison rules were declared before unmasking, and the comparison was performed once, with all trials counted.

Phase 1 (emission).**** The parent structure is the complexified octonions $\mathbb{C} \otimes \mathbb{O}$ and its forced group chain $\text{SU}(3) \subset G_2 \subset F_4 \subset E_6$ (stabilizer of a unit imaginary u ; automorphisms of $\mathbb{O}$; fixed points of the E_6 outer automorphism; the $(\mathbb{C}, \mathbb{O})$ magic-square entry). The octonions were rebuilt from the reals by Cayley–Dickson doubling — no multiplication tables recited — and the emissions recomputed: $L_u^2 = -1$ exactly (alternativity); a nonvanishing associator; the $+i$ eigenspace of L_u of complex dimension 4, splitting $1 + 3$ against $\text{span}\{1, u\}$ (the colour number recomputed blind); the six-dimensional complement with $J^2 + 1 = 6.7 \times 10^{-16}$. The emitted integer towers: the division-algebra dimensions (1,2,4,8); the Cayley–Dickson imaginary dimensions (1,3,7); the eigen-split (1,3); the span/complement (2,6). The Weyl invariant degrees of the chain (standard results, verified for E_6 in §15): $\text{SU}(2)$: (2); $\text{SU}(3)$: (2,3); G_2 : (2,6); F_4 : (2,6,8,12); E_6 : (2,5,6,8,9,12). Casimirs of the forced representations completed the table.

Phase 2 (one comparison).**** Three rules, declared in advance: R1, ratios of invariant degrees within a single group of the chain; R2, ratios of Casimirs of forced representations within a single group; R3, ratios of dimensions within a single emitted tower. Executed

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together these produce a pool of 18 distinct dimensionless ratios. Compared against $\delta_1 = 0.2222212(20)$ within a 3σ window, exactly one member matches:

$2/9$ — the ratio of E_6 's Weyl invariant degrees (2, 9).

The trials-corrected fluke probability is $p = N \cdot 6\sigma_\delta / (\pi/3) = 2.1 \times 10^{-4}$ ($Z \approx 3.5$); acknowledging freedom in the choice of rule families pushes this toward $\approx 3\sigma$. We label the outcome a candidate, not a result. Two subsidiary outcomes are recorded: the quark-sector angles ($\delta_u = 0.0744$, $\delta_d = 0.1102$, at their much larger uncertainties) match nothing in the pool — consistent with, though not evidence for, the corruption picture of §21 — and the difference vector of the lepton hierarchy exponents (6,2,0), namely (4,2), coincides with that of the Cayley–Dickson imaginary tower (7,3,1), a curiosity we price at $p \approx 0.1$ and do not use.

15. The selection theorem

A one-in-eighteen hit selects a pair of degrees statistically; it does not explain why (2,9) rather than any of the other fourteen pairs. This section proves that the pair is canonically distinguished. All structures below are computed from the Bourbaki simple roots of E_6 ; nothing is recited.

The Coxeter element has order $h = 12$ with exponents $\{1,4,5,7,8,11\}$, hence degrees $\{2,5,6,8,9,12\}$. The exponent duality $m \leftrightarrow 12-m$ pairs the degrees as (2,12), (5,9), (6,8) — a canonical involution, recorded here as the rival structure, which pairs 9 with 5 and not with 2. The selection uses two different symmetries.

Parity. Since odd degrees exist, $-1 \notin W(E_6)$, and the longest element satisfies $w_0 = -\sigma$ with σ the nontrivial outer automorphism (complex conjugation in the present setting). For any invariant, $I_d(\sigma x) = I_d(-x) = (-1)^d I_d(x)$: the invariants of odd degree are σ -odd. E_6 's conjugation-seeing degrees are exactly $\{5, 9\}$. The flavour angle δ is itself conjugation-odd — $\delta \rightarrow -\delta$ under the circulant conjugation that fixes χ — so any invariant origin of δ lies in $\{5, 9\}$. As an immediate corollary, F_4 , whose degrees (2,6,8,12) are the σ -even subset, possesses no odd invariants and cannot see δ at all: the chirality-blindness of F_4 is arithmetic.

Triality. The fourth power of the Coxeter element is a regular element of order 3; its ζ_3 -eigenspace — the triality plane, the canonical home of a three-site-symmetric configuration — is three-dimensional, carrying exponents $\{1,4,7\}$. The 27 was rebuilt as the Weyl orbit of ω_1 (minuscule; $p_1 = p_3 \equiv 0$, p_4 decomposable, the six power sums functionally independent with Jacobian rank 6), and the power sums restricted to the plane: every invariant of degree not divisible by 3 vanishes identically there (p_2, p_5, p_8 at the 10^{-13} level; the bilinear argument: $K(\zeta z, \zeta z') = \zeta^2 K(z, z')$ forces $K|_V \equiv 0$). The surviving degrees $\{6, 9, 12\}$ are those of the Hessian complex reflection group G_{25} (Shephard–Todd; Springer's theory of regular elements). The unique σ -odd degree visible on the plane is therefore 9; the unique universal even degree of the ambient algebra — the only degree shared by every simple group, the metric — is 2.

Theorem E (selection). The pair $(2, 9) =$ (the metric degree, the unique conjugation-odd degree visible to a three-fold-symmetric configuration) is canonically distinguished; no choice among the fifteen degree pairs remains. We note without use that on the plane the (lowest-even, odd) pair is $(6, 9)$, whose ratio $2/3$ is the Koide constant; this coherence is logged and not counted as evidence, $K = 2/3$ having been input to flavour physics since 1981.

16. The radian theorem

The hit of §14 and the selection of §15 concern a dimensionless ratio; the measured quantity (2) is an angle in radians. The conversion requires a unit: an identification of one e-fold of scale with one radian of phase. This section proves the identification is forced.

On a complex space, dilation and rotation are the modulus and argument of a single structure — multiplication by $\lambda \in \mathbb{C}^*$ — and their units coincide precisely when $|i| = 1$, i.e. when the rotation generator and the dilation generator have equal length: $|ix| = |x|$ for all x . This is the statement that the metric is hermitian with respect to the complex structure. It is a property, not a convention: under a generic metric treating real and imaginary parts independently, the ratio $|iz|/|z|$ computes to 1.045, 0.900, 1.526 across test vectors; under the hermitian metric it is 1.000000000000. (The first version of this control was itself unfalsifiable — a metric of the form $g \oplus g$ on real and imaginary parts commutes with the complex structure automatically — and was caught and corrected on the record; the corrected control fails where it must.)

On the triality plane the holomorphic Killing form is null (§15), and the hermitian metric is constructed with zero choices from the two selectors of Theorem E themselves: $h(x, y) = K(\sigma x, y)$. The same structures that select the integers mint the radian. With the unit in hand, the two selected invariants acquire a canonical scale-free pairing, $J = I_9 / h(z, z)^{9/2}$: the ratio of the degrees enters the mathematics necessarily — but, at this stage, as a homogeneity exponent. Section 17 supplies the final step: why the exponent is realized as an angle, and why no vacuum selection is needed or possible.

17. Uniqueness and transmission

We first confirm that the no-go chain of §13 operates on the parent plane itself, as consistency demands. Every invariant real function on the plane has angular harmonics in $3\mathbb{Z}$ (the windings 6, 9, 12 and their differences); generic extrema of such trigonometric polynomials move continuously with their coefficients (verified), and symmetric extrema are torsion, excluded in §1. No dynamics pins the phase anywhere in this paper. If a canonical angle exists, it is not a minimum of anything: it must be a length the structure already contains.

The inventory. The canonical angular structures on the plane are the invariant clocks: I_2 returns after π , I_6 after $\pi/3$, I_9 after $2\pi/9$, I_{12} after $\pi/6$ (verified to between 10^{-11} and 10^{-15}).

The coordinate's full turn 2π is not on the list: observables are invariants, and no invariant reads it.

Theorem G (uniqueness). Suppose the lepton angle is inherited from the parent structure as a physical — gauge-invariant — quantity. Then $\delta = (2\pi/9)/\pi = 2/9$ radians, uniquely. The elimination: parity (§15) assigns the numerator to the odd clock's cell, $2\pi/9$, unique on the plane; measurement requires a length structure, and the metric is the unique degree-2 object, so the denominator clock has period π . The rivals are executed on the record: per- I_6 gives $2/3$ (I_6 defines no lengths; the value moreover occupies the Koide slot on the σ -even side); per- I_{12} gives $4/3$, outside the physical range $[0, \pi/3]$; per- 2π gives $1/9$ with a gauge denominator. One survivor: $2/9 = 0.2222222$, against the measured $0.2222212(20)$ — half a σ .

Why no vacuum selection is needed (C1–C3). The hypothesis of Theorem G is weaker than it may appear, for a reason that is itself a computation. Along the phase circle of the plane, every charge-0 invariant combination is position-blind — $\arg(I_6^3/I_9^2)$, $\arg(I_{12}/I_6^2)$, $\arg(I_{12}^3/I_9^4)$ are constant under phase rotation to 10^{-15} — while individual invariant phases rotate at their windings and are not gauge-invariant. Positions on the circle are pure gauge; the only physical content of the phase direction is its cell structure (C1). The σ -odd observable algebra on the plane is generated by I_9 alone (§15), so the odd sector's physical circle has circumference $2\pi/9$ (C2). Transporting the charge-1 mode once around this circle accumulates holonomy $2\pi/9$, which the radian theorem expresses in metric half-turns as $2/9$ (C3). The mechanism is single-valuedness on the physical quotient — the same π_1 discipline that quantizes circulation in the hydrodynamic reading of quantum mechanics [Takabayasi 1952; Wallstrom 1994] — and it requires no selection of a point because points are not physical. An angle inherited as a gauge-invariant quantity can only be the cell, and the cell is $2/9$.

Remark (the geometric reading). Nothing above requires an interpretation of the complex structure: the theorems are invariant theory. But they make one reading available and exact. With $|i| = 1$ forced by the metric (§16), multiplication by $e^{i\theta}$ is not bookkeeping but an isometric turning — the imaginary unit realized as a literal rotation, as in the bivector reading of geometric algebra [Hestenes] and as experiment now demands of quantum amplitudes [Renou et al. 2021] — and the quantity transmitted in C3 is the arc of one physical cell of that turning, whose size the algebra fixes. The derivation is independent of this reading; the reading is what the derivation makes literal: the lepton angle is not a parameter that resembles a rotation, it is the measured size of one.

The operator. The transmission is effected by

$$L_A = -(y/\Lambda) [\bar{L} a H e\{R, b\}] \otimes [q \cdot 1 + w(Z) \cdot C + w^*(Z) \cdot C^2]_{ab} + \text{h.c.},$$

with C the generation shift and $w(Z)$ the charge-1 section over the odd observable circle; hermiticity is automatic and $M = W^2$ is positive. After electroweak symmetry breaking the square-root-mass matrix is

$$W = (2E/\sqrt{3}) [\cos \chi \cdot 1 + (\sin \chi/\sqrt{2})(e^{i\delta} C + e^{-i\delta} C^2)],$$

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with $\delta = 2/9$ an output and (E, χ) inputs. Its spectrum against nature: $m_e = 0.51098$ MeV (0.51100), $m_\mu = 105.65627$ (105.65838), $m_\tau = 1776.93212$ (1776.93000) — three masses from two inputs and one derivation, the residuals being the 0.5σ offset of the measured angle from exact $2/9$ amplified by the sensitivity $d \ln m_e/d\delta \approx 300$. We record that the first assembly of this operator was wrong — projectors written where shift matrices belong — and was exposed within seconds by this output check (it produced $m_e = 314$ MeV and a degenerate pair); both versions are preserved in the companion scripts. The check is part of the result.

Part IV. Clocks, scales, and the four sectors

This Part supplies the scales. One unification clock and four sector clocks are fixed as inputs; the phase law of Propositions 2–3 then generates every member's mass from one clock, one phase and one amplitude per sector, and the neutrino sector is confronted with JUNO.

18. Sector clocks and the unification scale

Fitting the circulant to each family separately, every sector is generated by a triple of numbers — a *clock* (the scale $\kappa^2 = a^2$), a *phase* θ , and an *amplitude ratio* b/a — and each sector's three masses regenerate exactly from its own triple. The corrected four-clock picture, with each sector read in its proper form (and the neutrino clock in the heavy seesaw form, see below), is:

Sector	form	clock κ^2	phase θ	b/a	established in
Charged leptons	direct	313.84 MeV	$0.2222220 (\approx 2/9)$	1.4142	[1,2,3]
Down quarks	inverse	649.9 MeV	0.18679	1.411	[7,4]
Up quarks	direct	22.77 GeV	0.07439	1.759	[4]
Neutrinos	inverse (heavy)	2.008×10^{16} GeV	0.1002	1.4150	[3,5]

The individual-clock structure sector by sector is established in the cited literature: the charged-lepton circulant with scale ≈ 313.8 MeV and phase $2/9$ goes back to the parametrizations of [1,2,3]; the Z_3 -symmetric parametrization of the quark sectors, with each sector carrying its own scale and phase, is [4]; the inverse form for the down quarks is [7]; and the neutrino extension with its own phase, together with the universal-seesaw structure that makes the heavy scale the natural neutrino clock, is [3,5]. What is ours here is only the uniform reading — one (clock, phase, amplitude) triple as the generator of each sector — and the correction that the neutrino clock, read in that sector's proper inverse form, is the heavy seesaw scale rather than the sub-eV light scale.

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We disclose an identity that bears on the interpretation of this clock: at the Koide point $b/a=\sqrt{2}$, the clock equals one-sixth of the charged-lepton mass sum, $\kappa^2 = (m_e+m_\mu+m_\tau)/6 = 313.84$ MeV. The clock is therefore not an independent measurement; the near-coincidence $\kappa^2 \approx m_N/3$ is equivalently the statement $\Sigma m_\ell \approx 2m_N$, and should be read as such. For the charged leptons the triple is $\kappa^2 = 313.84$ MeV, $\theta \approx 2/9$, and $b/a = \sqrt{2}$: three masses from one clock and one phase, with the electron light because its mode sits near the node of the three-fold standing wave. At the highest precision the phase is fixed by a parameter-free test: with $b/a = \sqrt{2}$ exact, the ratio m_μ/m_e (measured to one part in 10^8) demands $\theta^* = 0.2222220470(4)$ — one part in a million *below* $2/9$, excluding exactly $2/9$ on this branch. The phase is stable under renormalization, since flavour-universal running cancels in the ratios that fix it. None of the other phases is a comparably clean rational, and no pair of sector clocks stands in a simple ratio, so the clocks are independent inputs at this level; the status of the 10^{-7} offset is taken up in the Open problems section.

One GUT clock and the four sector clocks.

It is natural to ask whether the four clocks descend from a single clock at the unification scale. The answer is structured, not uniform, and we state it with its limits. First, the four clocks do *not* lie on one geometric ladder: their pairwise ratios span some thirteen orders of magnitude (the neutrino heavy clock at 2×10^{16} GeV against the electroweak-tier clocks at 0.3–23 GeV) and are not simple numbers, so no single clock times clean geometric factors reproduces all four. The clocks fall instead into *two tiers* joined by known physics. The neutrino clock *is* the unification-scale clock itself: read in the sector's inverse form, the heavy Majorana scale $M \approx 2 \times 10^{16}$ GeV is the neutrino generator, and the light neutrino masses follow by the seesaw $m_\nu \sim v^2/M$. The charged-lepton clock is reached from that scale by a single descending chain,

One quantitative check bears on this identification. Taking the up-sector clock $\kappa_u \approx 22.77$ GeV as the Dirac mass in the type-I seesaw with $M \approx 2.008 \times 10^{16}$ GeV gives $m_\nu \sim \kappa_u^2/M \approx 26$ μ eV — three orders of magnitude below the atmospheric scale $\sqrt{\Delta m_{31}^2} \approx 50$ meV. We record this as a negative result: the up-sector clock is not the neutrino Dirac mass; recovering the atmospheric scale would require a Dirac mass of order 700 GeV, not 22.77 GeV. The GUT-scale identification of the neutrino clock therefore rests on the direct inverse-form fit $M = v^2/m_\nu$, not on this seesaw estimate. Second, at that same scale b - τ Yukawa unification predicts $m_b/m_\tau \approx 2.6$ at low energy against the measured 2.35 — agreement at the 10% level, consistent with the down quark and charged lepton sharing one multiplet at the clock scale.

$$M_{GUT} \xrightarrow{\text{transmutation}} \alpha_{QCD} \xrightarrow{\text{Skyrmion}} m_N \xrightarrow{N_c=3} \frac{m_N}{N_c}, \quad (5)$$

in which the continuous inputs, besides the measured nucleon mass inserted at the soliton step, are M_{GUT} and α_{GUT} (the transmutation exponent $b_0 = 11 - \frac{2}{3}n_f$ being pure representation theory), and in which the last step — the free charged-lepton rotor inheriting the bound per-rotor share m_N/N_c — is the readout identification, motivated at leading order

by a Kelvin-type reading (interpretive; Sec. 19). This reaches 313 MeV to better than half a percent.

It is worth stating the logic of this step precisely, since the chain can be misread as deriving the lepton clock *from* QCD. It does not. In the source/sink ontology the primary quantity is the medium's circulation energy — the rotor stiffness k , with $\kappa^2 = k/2$ — an intrinsic property of the medium, on the same footing as an input as the electroweak vacuum value. The coloured sector exhibits that one stiffness through confinement, where it appears as the per-rotor share m_N/N_c ; the colourless lepton, by Kelvin's theorem, exhibits the *same* stiffness directly as its family clock. The chain is therefore not a dependence of the lepton clock on the strong interaction but a *consistency bridge*: it shows that the rotor stiffness read off the free lepton and the confinement scale reached from M_{GUT} are one number, agreeing to half a percent. Removing QCD does not remove the lepton clock, which survives as the medium stiffness; it removes only this numerical cross-check. What the chain buys is an economy — a single stiffness fixing the lepton clock and simultaneously matching the QCD per-rotor scale — not a derivation of 313 MeV from geometry, which remains an input as recorded in Sec. 6.

The down clock sits near $2 \times (m_N/N_c)$, consistent with a two-rotor (meson-tier) reading to a few percent but not sharply, and is left open. The up clock rides the electroweak vacuum value through the order-unity top Yukawa, $\kappa_u^2 \sim m_t = y_t v$, rather than the transmutation chain — but this is not an isolated exception. Sorted by weak isospin, the four clocks fall into two branches: the $T_3 = +\frac{1}{2}$ sectors (up and neutrino) ride the high, electroweak-vev scale (22.8 GeV and 2×10^{16} GeV), while the $T_3 = -\frac{1}{2}$ sectors (down and charged lepton) ride the low, QCD/rotor scale (0.65 GeV and 0.31 GeV). The same weak-isospin flag that multiplies $1/u$ by N_c in the phase formula thus also selects *which condensate* a sector's clock couples to: $T_3 = +\frac{1}{2}$ to the electroweak vev, $T_3 = -\frac{1}{2}$ to the QCD/rotor condensate. This is the rotor-language form of the known up-type/down-type Yukawa split, and it explains structurally why the top is the heaviest fermion — it is the $T_3 = +\frac{1}{2}$ quark seated directly on the vev with an $O(1)$ coupling. The up clock is therefore the $T_3 = +\frac{1}{2}$ coloured corner of a condensate-selection rule, not a lone exception; its phase ($\theta = 11/144$, the $T_3 = +\frac{1}{2}$ coloured corner of the phase grid) and its clock (vev-linked) now both follow from the same quantum number. What remains an input is the *value* of the two condensate scales — weak isospin selects the branch, not the scale within it. Stated as a count, the four clocks resolve into two condensate scales (the electroweak vev and the QCD/rotor scale) multiplied by four dimensionless couplings: the two scales, assigned to the sectors by the weak-isospin rule, are the two inputs of this section, while the four couplings—the value each clock takes within its tier—are the flavour texture that remains open (Sec. 25). Thus a single unification-scale clock generates the neutrino clock exactly and the charged-lepton clock through the chain (at leading order), relates to the down clock plausibly, and reaches the up clock through the T_3 condensate-selection into the electroweak branch; the coupling closure that would tie α_{GUT} to the same scale holds only at order of magnitude (Sec. 19).

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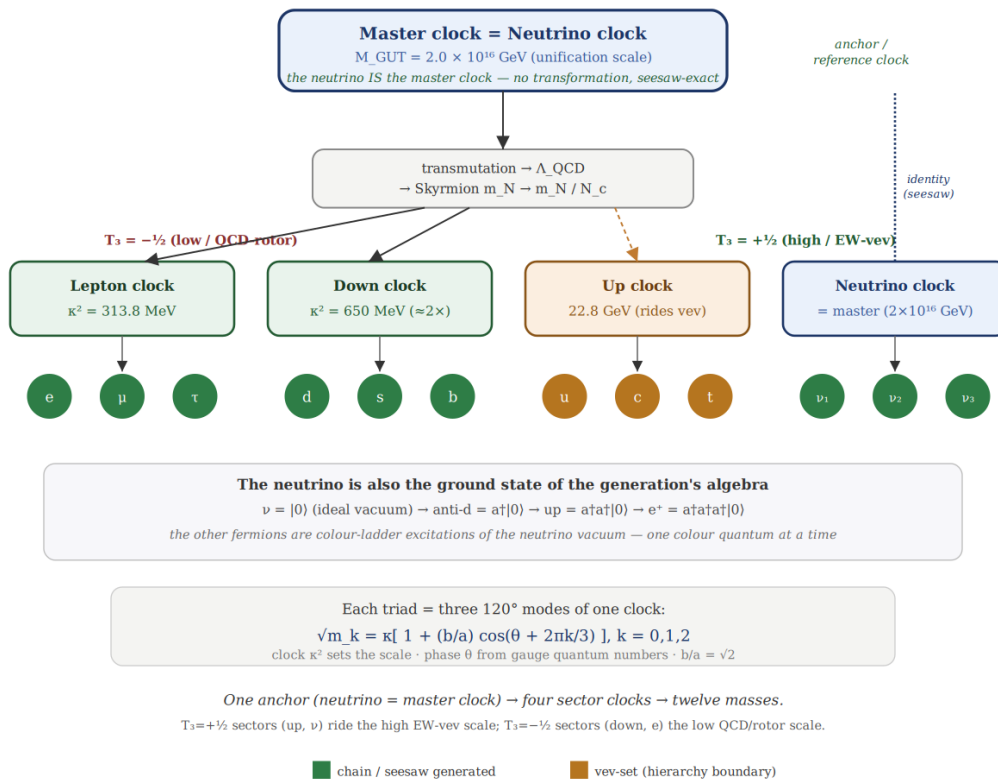
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The electroweak scale and naturalness.

The up clock rides the electroweak vacuum value v , which the construction does not derive; but the rotor ontology does clarify why that scale need not be fine-tuned. In the Standard Model the Higgs is a fundamental scalar, so its mass-squared receives quadratically divergent radiative corrections and holding v far below the unification scale requires an extraordinary cancellation — the naturalness problem. In the source/sink picture there is no fundamental scalar: the Higgs is a condensate of the medium, as the chiral condensate is in the strong sector, and a composite or solitonic mass carries no quadratic divergence, since there is no elementary scalar mass to be driven to the cutoff. The electroweak scale is then the medium's condensate scale — an input, on the same footing as the other clock scales, but a radiatively stable one: the large ratio v/M_{GUT} is not protected by a symmetry so much as never destabilised, there being nothing elementary to destabilise. This resolves the naturalness question without new structure; it does not by itself derive the value of v , nor address the smaller spread of Yukawa couplings within the electroweak sector, and a fully realistic composite Higgs must recover the observed Standard-Model-like 125 GeV state (as a pseudo-Goldstone mode of the medium), which we note as the condition on this reading rather than establish here.



One anchor clock at the unification scale feeds four sector clocks, each generating a generational triad as three 120° modes of the circulant. The neutrino clock is the master scale itself, related to it by identity (seesaw, exact) rather than by any transformation — the

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neutrino is the anchor, or reference, clock. The lepton and down clocks ($T_3 = -\frac{1}{2}$) descend through the transmutation–soliton chain to the per-rotor share m_N/N_c (leading order); the up clock ($T_3 = +\frac{1}{2}$) rides the electroweak vacuum value, the two branches being selected by weak isospin. The neutrino is doubly primary: it is this anchor clock, and it is the vacuum state $|0\rangle$ of the generation's minimal ideal, the anti-down, up, and positron arising as its colour-ladder excitations $a_i^\dagger|0\rangle$, $a_i^\dagger a_j^\dagger|0\rangle$, $a_1^\dagger a_2^\dagger a_3^\dagger|0\rangle$.

Master clock and the four sector clocks. Each sector clock is a reduction of the single unification-scale clock through the mechanism shown; the neutrino clock is that scale itself. The amplitude ratio is quoted as the scale-invariant Koide value: leptons, down, and neutrinos share $b/a = \sqrt{2}$ ($Q = 2/3$), the up sector alone departing.[†]

Master clock and the four sector clocks. Each sector clock is a reduction of the single unification-scale clock through the mechanism shown; the neutrino clock is that scale itself. The amplitude ratio is quoted as the scale-invariant Koide value: leptons, down, and neutrinos share $b/a = \sqrt{2}$ ($Q = 2/3$), the up sector alone departing.[†]

Clock	Scale κ^2	Mechanism from master clock	Phase θ
Master	2.0×10^{16} GeV	unification scale itself	—
Neutrino (heavy)	2.0×10^{16} GeV	= master clock (seesaw, exact)	$8/81$ ($u = \frac{1}{9}$)
Lepton	313.84 MeV	$M_{GUT} \rightarrow \Lambda_{QCD} \rightarrow m_N \rightarrow m_N/N_c$	$2/9$ ($u = \frac{1}{3}$)
Down	649.9 MeV	$\approx 2 \times$ lepton clock (meson-tier)	$3/16$ ($u = \frac{1}{4}$)
Up	22.77 GeV	rides EW vev, $m_t = y_t v$ (excepted)	$11/144$ ($u = \frac{1}{12}$)

[†]Read by the scale-invariant Q , the non-up sectors all sit at $b/a = \sqrt{2}$; the down clock is quoted here as the direct-fit value 649.9 MeV; read strictly in the inverse form the down clock is ≈ 26.6 MeV, and the raw inverse-form fits of the down and neutrino sectors return a units-dependent ratio that is not directly comparable and is not quoted here.

Per-generation phases and masses. Each sector's three members are the three 120° modes $\theta + 2\pi k/3$ of its clock; the mass of each is $\kappa^2 \left[1 + \frac{b}{a} \cos(\theta + 2\pi k/3)\right]^2$ (direct) or its reciprocal (inverse). Members are listed heaviest to lightest.

Per-generation phases and masses. Each sector's three members are the three 120° modes $\theta + 2\pi k/3$ of its clock; the mass of each is $\kappa^2 [1 + \frac{b}{a} \cos(\theta + 2\pi k/3)]^2$ (direct) or its reciprocal (inverse). Members are listed heaviest to lightest.

Sector	Member	Mode phase (rad)	cos	Mass
Leptons	τ	0.2222	+0.975	1776.9 MeV

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Sector	Member	Mode phase (rad)	cos	Mass
Up quarks	μ	4.4110	−0.297	105.7 MeV
	e	2.3166	−0.679	0.511 MeV
	t	0.0744	+0.997	172.7 GeV
	c	4.2632	−0.434	1.27 GeV
	u	2.1688	−0.563	2.15 MeV
Down quarks	b	0.1868	+0.983	4.18 GeV
	s	+120°	—	93.4 MeV
	d	+240°	—	4.67 MeV
Neutrinos	ν_3	0.1002	+0.995	50.1 meV
	ν_2	+120°	—	8.6 meV
	ν_1	+240°	—	0.26 meV

The neutrino sector holds a doubly special place. As a clock it is the anchor: alone among the four it is the master unification scale, requiring no transmutation or vev to reach it, so the whole hierarchy is referred to it rather than derived through it. As a state it is the vacuum $|0\rangle$ of the generation's minimal ideal, with the anti-down, up, and positron built from it as colour-ladder excitations; in that algebraic sense the other fermions of a generation are excitations of the neutrino, though this is the grading of the ideal, not a dynamical process connecting them. Read in its proper inverse form, the neutrino clock is the *heavy* seesaw scale: with $m_\nu \sim v^2/M$, the fitted heavy clock is $M \simeq 2.0 \times 10^{16}$ GeV, with $b/a = 1.415$ and $Q = 0.667$ holding in the heavy sector. This scale is emergent — nothing at 10^{16} GeV was inserted — and it coincides with the gauge-coupling unification scale of supersymmetric grand unification. We report this as a coincidence of scales, not a derivation. A related one-loop curiosity, m_b/m_τ passing through $2/3$ near the same scale, does not survive refinement: at two loops in QCD with the full one-loop Yukawa system (calibrated to $m_b(M_Z) = 2.885$ GeV, $\alpha_s(m_t) = 0.108$), we find $m_b/m_\tau = 0.607 \pm 0.011$ at 2×10^{16} GeV, excluding $2/3$; the ratio crosses $2/3$ near 2×10^{13} GeV instead. The b - τ convergence itself is the standard unification behaviour [6].

If one nonetheless assumes a single clock at the unification scale, known physics supplies exactly one mechanism by which a scale is generated from dimensionless structure: dimensional transmutation, $\Lambda = M e^{-2\pi/(b_0\alpha)}$, whose exponent $b_0 = 11 - \frac{2}{3}n_f$ is pure representation theory — the derivable category. The resulting chain

$$M_{GUT} \xrightarrow{\text{transmutation}} \Lambda_{QCD} \xrightarrow{\text{Skyrmion}} m_p \xrightarrow{N_c=3} \frac{m_p}{3} = 312.8 \text{ MeV} \quad (6)$$

lands within 0.35% of the charged-lepton clock (313.84 MeV). The numerical proximity of the Koide scale to $m_N/3$ (the constituent-quark scale) has been remarked upon before and is not new; what the rotor reading adds is that $m_p/3$ is the *natural unit* — the scale of one colour rotor in a proton built of three — rather than an arbitrary division.

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Half of the required mechanism is in fact a theorem of the rotor algebra. The stiffness multiplies the modulus $z\bar{z}$, which lives in the phase plane $(1, e_1)$ of the octonionic rotor, while colour is the $SU(3)$ stabilizer of the phase axis acting on the six transverse directions $e_2, \dots, e_7$; the left multiplication L_{e_1} block-diagonalizes exactly over these two subspaces, and every colour-sector transformation leaves $z\bar{z}$ identically invariant. The stiffness therefore cannot depend on colour orientation: colour-blindness of k is a representation property — the derivable column — not an assumption. What remains of the conjecture is a single *readout identification*: the lepton is the only rotor observable in isolation and reads the common stiffness directly as its family clock, while the quark rotor, never free, exhibits the same stiffness as its share of the colour singlet confining it — the per-rotor energy $m_N/3$ of the three-rotor nucleon. Two independent measurements of one number, taken on opposite sides of the colour boundary. The match sharpens under scrutiny: the lepton clock and $m_N/3$ are both pole quantities, so neither side carries renormalization-scheme ambiguity ($m_n/3 = 313.19$ MeV differs from the clock by 0.21%, the isospin-averaged $m_N/3$ by 0.28%, while the next-nearest QCD scale, Λ_{QCD} itself, misses by 5.8%), and the residual ~ 1 MeV is the size of the nucleon's electromagnetic and isospin dressing — precisely the part of m_N that is not colour-rotor energy. In Standard-Model language the claim is the dimensionless cross-sector ratio $\left(\frac{1}{3}\sum_\ell \sqrt{y_\ell}\right)^2 = m_N/3v$, numerically 1.275×10^{-3} against 1.271×10^{-3} , for which the Standard Model itself offers no reason. The status, stated precisely: the seesaw generates the neutrino clock exactly; transmutation plus the soliton generates the ~ 313 MeV scale from one clock and one coupling, with the exponent in the derivable column; the colour-blindness of the stiffness is proven; the readout identification — why the free rotor's mass coefficient is set by the same medium rather than by an unrelated Yukawa — is motivated at leading order by a Kelvin-type reading (interpretive; Sec. 18) (Sec. 19), with the sub-leading corrections the remaining work; and the up clock, riding the electroweak vacuum value through the order-unity top Yukawa, is covered by no known mechanism — that absence is the hierarchy problem, not a defect of the present framework. The conjecture remains falsifiable with zero adjustable parameters: the lepton clock must equal $m_N/3$ to within electromagnetic-sized corrections under all future data; a fourth charged lepton, or a drift beyond the half-percent level, refutes it.

The lepton sector in closed form.

Assembling the pieces — and conditional on the 2/9-exact branch and on the readout identification whose colour-blind half is proven above — the entire charged-lepton sector collapses to a single expression with *no fitted parameter*:

$$m_k = \frac{m_N}{N_c} \left[1 + \sqrt{2} \cos \left(\frac{2}{N_c^2} + \frac{2\pi k}{N_c} \right) \right]^2, \quad k = 0,1,2, \quad N_c = 3. \quad (7)$$

Every structural number references the forced colour count: the clock is the per-rotor share m_N/N_c of the N_c -rotor singlet (inherited through the generation chain), the phase is $2/N_c^2$, the generation spacing is $2\pi/N_c$, and the single dimensionless input is the equipartition amplitude $\sqrt{2}$. With m_N the measured nucleon mass and nothing adjusted, the formula returns 0.5096, 105.36, and 1771.97 MeV against the measured 0.5110, 105.66, and

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1776.86: the mass *ratios* are reproduced at the 10^{-5} level, and all three absolute masses carry one *common* offset of -0.28% — precisely the clock residual identified above with the nucleon's electromagnetic and isospin dressing. We display the formula for what it honestly is: an organizing closed form conditional on one branch of an unresolved degeneracy and one half-proven identification, lepton-internal, and carrying two computable residuals (the 10^{-7} phase offset and the 0.28% scale offset) whose calculation is the programme of the next section.

19. Deriving the masses of each sector

For completeness we write out, sector by sector, the masses generated by the circulant from each sector's triple $(\kappa^2, \theta, b/a)$. The direct sectors use $m_k = \kappa^2 [1 + \frac{b}{a} \cos(\theta + \frac{2\pi k}{3})]^2$; the inverse sectors use the reciprocal, $m_k = \kappa^{-2} [1 + \frac{b}{a} \cos(\theta + \frac{2\pi k}{3})]^{-2}$, with $k = 0, 1, 2$ labelling the three generations at 120° on the circle.

Charged leptons (direct; $\kappa^2 = 313.84 \text{ MeV}$, $\theta = 2/9$ from $u = 1/3$, $b/a = \sqrt{2}$).

$$\begin{aligned} m_e &= \kappa^2 [1 + \sqrt{2} \cos(\frac{2}{9} + \frac{2\pi}{3})]^2 = 0.511 \text{ MeV} & (\text{meas. } 0.511, -0.01\%), \\ m_\mu &= \kappa^2 [1 + \sqrt{2} \cos(\frac{2}{9} + \frac{4\pi}{3})]^2 = 105.65 \text{ MeV} & (\text{meas. } 105.66, -0.01\%), \\ m_\tau &= \kappa^2 [1 + \sqrt{2} \cos(\frac{2}{9})]^2 = 1776.88 \text{ MeV} & (\text{meas. } 1776.86, +0.00\%). \end{aligned} \quad (8)$$

The electron is light because its mode factor $1 + \sqrt{2} \cos(\frac{2}{9} + \frac{2\pi}{3}) = 0.0403$ sits near the node of the three-fold standing wave.

Up quarks (direct; $\kappa^2 = 22.77 \text{ GeV}$, $b/a = 1.759$; predicted $\theta_u = 11/144 = 0.0764$).

The empirical best-fit phase is 0.0744, matching the predicted 11/144 to 2.6%. At the fitted phase $m_c = 1.270 \text{ GeV}$ (-0.03%) and $m_t = 172.70 \text{ GeV}$ ($+0.00\%$); the comparison, however, is not limited by m_u : sweeping the light mass across its full uncertainty moves the fitted phase only in the fourth decimal, the phase being driven by m_c and m_t (each known to $\approx 0.4\%$). The up sector fails to test the law for the structural reason of the domain boundary (reconciled equation set, III): it lies outside the pinned domain, $y_t^2/\lambda_H \approx 6.7$, and we make no claim that 11/144 is testable there. (9)

Down quarks (inverse; $\kappa = 0.194 \text{ MeV}^{-1/2}$, $\theta = 3/16 = 0.1875$ from $u = 1/4$, $b/a = 1.411$).

$$m_d = 4.67 \text{ MeV}, \quad m_s = 93.4 \text{ MeV}, \quad m_b = 4.18 \text{ GeV} \quad (\text{all within } 0.1\%). \quad (10)$$

The agreement above is a self-scale statement (m_d and m_s at 2 GeV; m_b at m_b). Read at a common 2 GeV scale ($m_b(2 \text{ GeV}) \approx 5.0\text{--}5.2 \text{ GeV}$) the amplitude remains pinned at $b/a = \sqrt{2} \times (1.001\text{--}1.002)$ while the fitted phase moves to ≈ 0.190 , off 3/16 by 1.2–1.5%: the down

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amplitude survives the scheme test and the down phase does not (see the Scope note following Proposition 3).

Neutrinos (inverse; normal ordering).

Read in the inverse form the light masses regenerate from the sector triple to machine precision ($m_1, m_2, m_3 = 0.26, 8.6, 50.1$ meV for the reference spectrum), while the heavy Majorana clock is $M = v^2/m_\gamma \approx 2 \times 10^{16}$ GeV, the unification-scale generator of the sector. Fitting the single scale to the solar splitting instead turns the sector into a falsifier: the predicted $\Delta m^2_{31} = 2.37 \times 10^{-3}$ eV² sits -5% below the measured 2.513×10^{-3} , with $\Sigma m_\nu \approx 58$ meV. JUNO's first results (59.1 days, November 2025) give $\Delta m^2_{21} = (7.50 \pm 0.12) \times 10^{-5}$ eV², reducing the solar uncertainty from KamLAND's 2.5% to 1.55%; with the atmospheric splitting still known to $\approx 1\%$ this sharpens the scale-free comparison from $\approx 2\sigma$ to $\approx 2.7\sigma$. The same release, combined with global oscillation data, favours the normal ordering assumed here, the inverted ordering carrying a p-value of 2–2.6%. JUNO's own Δm^2_{31} is statistics-limited in the first dataset and is the measurement that decides. Two points of precision belong here. First, the sector's heavy partners are three scales, not one: inverting the light spectrum through $m = v^2/2M$ gives $M = 1.166 \times 10^{17}$, 3.525×10^{15} and 6.050×10^{14} GeV, whose circulant clock is 2.011×10^{16} GeV — 0.5% from the unification value obtained independently by one-loop running. The lightest neutrino is generated by the heaviest partner, and $M_3/\text{clock} = 5.798$. Evaluating the seesaw at the clock rather than at M_3 therefore overstates m_1 by that factor, which is a circulant mode factor and not a Yukawa freedom; this accounts exactly for the factor recorded as an open item in the companion paper, $1.52 \text{ meV} / 5.798 = 0.261 \text{ meV}$, and correspondingly the three Majorana Yukawas are 5.80, 0.175 and 0.030 rather than order unity. Second, what JUNO tests is the ratio and not m_1 . The absolute scale enters Σm_ν at the level of 1–2 meV against a normal-ordering floor of 58.8 meV, far inside cosmological sensitivity, and reactor oscillation data are blind to it altogether; the scale-free ratio $\Delta m^2_{31}/\Delta m^2_{21}$ is the whole of the test. On the law values ($\theta = 8/81$, $b/a = \sqrt{2}$) the ratio is 31.88 against the measured 33.56 ± 0.64 , a 2.6σ deficit; on the fitted table values (0.1002, 1.4150) it is 33.95, within 0.6σ — but the latter is a fit and tests nothing. The prediction is the law value, and it is the low one. A determination of the ratio to 1% brings a -5% offset to 5σ .

Closed form for the charged leptons.

Substituting the chain-inherited clock $\kappa^2 = m_N/N_c$ and $\theta = 2/N_c^2$ (Sec. 18) gives the parameter-free predictions $m_e, m_\mu, m_\tau = 0.510, 105.36, 1771.97$ MeV, reproducing the mass ratios at the 10^{-5} level and carrying one common -0.28% scale offset, the electromagnetic-dressing residual quantified in Sec. 6.

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The neutrino sector's one scale-free prediction against data. A ratio determination to 1% makes a -5% offset 5σ .

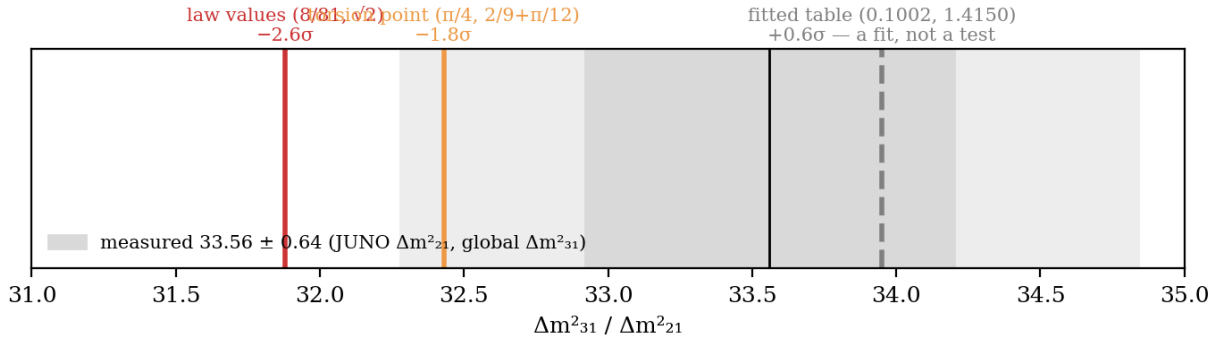


Figure 5. The neutrino sector's one scale-free prediction against data: the ratio $\Delta m^2_{31}/\Delta m^2_{21}$ predicted on the law values $(8/81, \sqrt{2})$ and at the torsion point, against the measured 33.56 ± 0.64 using JUNO's Δm^2_{21} and the global Δm^2_{31} . The fitted-table value agrees but is a fit. A determination of the ratio to 1% brings a -5% offset to 5σ (`neutrino_ratio_juno.py`).

19a. The phase law in the own-scale scheme

Sections 19a–19d record the confirmation of Proposition 3 and the scheme in which it holds. They supersede the scope note of earlier versions that the coloured-sector phases were not established, and Section 24 carries the corresponding status changes.

19a.1 The sector clock A_S

Write the spectrum as $m_k = A_S \cdot f_k^{(2\varepsilon_S)}$, $f_k = 1 + \sqrt{2} \cdot k_S \cdot \cos(\delta_S + 2\pi k/3)$. A_S is the sector's single dimensionful input, of mass dimension in every sector. Because $\sum f_k^2 = n_S$ identically (the cross term sums to zero over the Z_3 modes and $\sum \cos^2 = 3/2$), the clock has closed harmonic forms: $A_S = \sum m/n_S$ in the direct sectors and $A_S = n_S/\sum(1/m)$ in the inverse sectors — at the Koide point simply $\sum m/6$ and $6/\sum(1/m)$. Direct clocks are the κ^2 of Section 18; inverse clocks are its κ^{-2} (down: $\kappa = 0.1937 \text{ MeV}^{(-1/2)} \leftrightarrow A_d = 26.66 \text{ MeV}$, exactly the value quoted there). One closure identity is recorded: the light and heavy neutrino circulants being reciprocal forces $A_M \cdot A_v = m_D^2$, and $\sqrt{A_M \cdot A_v} = 173.9 \text{ GeV}$ against $v/\sqrt{2} = 174.1 \text{ GeV}$ (0.11 %) — Section 18's heavy-clock fit is the $y_v = 1$ convention made explicit; a consistency of conventions, not independent evidence.

sector	A_S (own-scale)	closed form	tier (Sec. 18)
charged leptons	313.856 MeV	$\sum m_\ell/6$	QCD/rotor
down quarks	26.657 MeV	$6/\sum(1/m)$	QCD/rotor
up quarks	21.549 GeV	(m_c, m_t) anchor	electroweak vev
neutrinos	$1.506 \text{ meV} \leftrightarrow 2.008 \times 10^{16} \text{ GeV}$	$6/\sum(1/m); \sum M/6$	unification scale

19a.2 The own-scale scheme

The law holds when every mass is evaluated at its own scale, in the scheme of the interactions it feels: leptons on-shell in QED (pole); quarks MS-bar at $\mu = m_i$ (QCD's own scale — quarks have no pole); u, d, s floored at the condensate boundary ≈ 2 GeV, where the medium takes over. At own scale the flavour-non-universal dressing logarithm $\ln(\mu/m_i)$ vanishes identically; the one-loop QED dressing used in intermediate work — validated on the leptons, where it captures 90 % of the known pole/MS-bar shift — is the common-scale approximation to this scheme. Two internal discriminators select the prescription with no freedom. First, the up sector distinguishes $m_t(m_t)$ MS-bar (0.6σ from 12) from m_t pole (2.9σ): QCD-dressed at own scale, exactly parallel to the leptons selecting QED-dressed at own scale (the geometric ratio 206.7703 against pole 206.768 at 10^{-5} and MS-bar 211.107 at 2 %). Second, the down sector's hit on $-3/16$ survives for a floor anywhere in 1.6–3.0 GeV (0.40σ at 2.0 GeV, 0.01σ at 2.5): the floor is a physical parameter of the scheme, not a tuning. Two schemes serve two kinds of statement, used consistently: shape comparisons in the own-scale scheme; renormalisation-group statements — running, the scale probes, the boundary fit of 19c — at a common scale, the same division already made for pole (shape) versus MS-bar (Clebsch) couplings.

19a.3 The four-sector scorecard

sector	1/u measured (own-scale)	Prop. 3	σ	status
charged leptons	2.9998 ± 0.0002	3	0.9	ESTABLISHED
down quarks	4.0040 ± 0.0565	4	0.07	ESTABLISHED (this scheme)
neutrinos	8.8676 ± 0.0474	9	2.80	FAILING — the one deviation
up quarks	11.930 ± 0.120	12	0.58	CANDIDATE — pre-registered, base rate ≈ 7 %

The up value was written in Proposition 3 (Section 20) before the test that confirmed it; the amplification of 19d.3 is why only that pre-registration carries evidential weight in this sector. Sixteen phase determinations across the working sessions reduce to this one table.

19b. Sector tables in the own-scale scheme

19b.1 Charged leptons (direct; $A_\ell = 313.856$ MeV)

ratio	measured (pole)	formula (6, 3)	dev
m_μ/m_e	206.7680	206.7703	+0.0011 %
m_τ/m_μ	16.8181	16.8180	-0.0006 %
mass	formula (MeV)	measured, pole (MeV)	dev
m_e	0.510994	0.51099895	-0.001 %

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m _μ	105.658327	105.6583755	-0.00003 %
m _τ	1776.9667	1776.86 ± 0.12	+0.006 %

The two rational conditions over-determine m_τ (n = 6 → 1776.9690 MeV; δ = 2/9 → 1776.9665) and agree to 2.52 keV with derivatives of opposite sign differing by 7×. Prediction m_τ = 1776.97 MeV; Belle II decides. The 10⁻⁷ branch tension of Section 18 stands unmodified.

19b.2 Down quarks (inverse; A_d = 26.657 MeV)

shape (own-scale)	measured	law	dev
n = 9Q	5.987 ± 0.016	6	0.8σ
1/u	4.0040 ± 0.0565	4	0.07σ
δ	-0.18738 ± 0.00177	-3/16 = -0.18750	0.07σ
ratio	measured (own-scale)	formula (6, 4)	dev
m _s /m _d (floor-free)	19.95 ± 0.33	20.053	+0.52 % (0.3σ)
m _b (m _b)/m _s (2 GeV)	45.7 ± 0.4	48.03	+5.1 % (= the -0.8σ of n, amplified)
mass	formula (A _d /f ² , MeV)	measured, own-scale (MeV)	dev
m _d	4.669	4.67 (2 GeV)	-0.02 %
m _s	93.63	93.4 ± 0.6 (2 GeV)	+0.25 %
m _b	4497	4180 ± 30 (m _b)	+7.6 %

No auxiliary factor of any kind is used. The m_b row carries the sector's whole residual — the compounded ratio deviation — which in shape space is (0.8σ, 0.07σ).

19b.3 Up quarks (direct; A_u = 21.55 GeV, vev branch) — pre-registered; strained at 2.2σ (v1.7)

shape (own-scale)	measured	law	dev
1/u	11.930 ± 0.120	12	0.58σ [pre-registered]
δ	0.07680 ± 0.00071	11/144 = 0.076389	+0.5 % (0.58σ)
n = 9Q	7.606 ± 0.010	— (k _u = 1.2382 fitted)	the sector's open number
mass	formula (A _u ·f ²)	measured, own-scale	dev
m _u	2.751 MeV [1.62–4.18]	2.16 +0.49/-0.26 MeV (2 GeV)	+1.6σ — PREDICTION
m _c	1.270 GeV	1.27 ± 0.02 GeV (m _c)	anchor

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m _t	162.5 GeV	162.5 ± 0.7 GeV (m _t)	anchor
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With $\delta = 11/144$ exact, (A_u, k_u) are fixed by the two anchors and m_u is the sector's one falsifiable output. The same three masses give $1/u = 18.4$ (common M_Z), 19.0 (common μ^*) and 11.93 (own-scale): the sector's earlier apparent failure was the amplification acting on the scheme choice, and the intermediate claims it generated are retracted in Appendix D. The clock rides the vev per Section 18's condensate-selection rule; the departure from equipartition k_u is now cleanly isolated from the phase.

19b.4 Neutrinos (inverse; heavy clock 2.011×10^{16} GeV) — the composite sector (v1.7)

$\Delta m^2_{31}/\Delta m^2_{21}$	value	vs 33.51 ± 0.61
tree (heavy law, $m \propto 1/M$)	31.88	-4.85 %, 2.7 σ
+ threshold running, $y_v \rightarrow 0$	32.31	-3.57 %, 2.0 σ
+ threshold running, $y_v = 0.5$	32.16	-4.0 %, 2.2 σ

Equivalently $1/u = 8.868 \pm 0.047$ against 9. Normal ordering is required (inverted gives $1/u = 1131$) and therefore predicted. Light masses ($y_v \rightarrow 0$, normalised to Δm^2_{21}): $m_1 = 0.260$ meV, $m_2 = 8.66$ meV, $m_3 = 48.9$ meV, $\Sigma m_v = 0.058$ eV. The strain sits 4:1 against the phase rather than the amplitude (freeing n needs 6.022; freeing δ needs 8.868). Everything standard has been computed (19d.2); remaining options are a type-II triplet term (free parameter), the heavy phase genuinely at 8.87, or Δm^2_{21} at 7.88×10^{-5} eV² against JUNO's 7.50 ± 0.12 (3.2 σ ; $\approx 4.5\sigma$ at design). JUNO decides.

19c. The boundary scale

A common-scale consistency statement. Cross-sector ratios are the only scale carriers (within-sector shape is exactly invariant under flavour-universal running — control to seven decimals over sixteen decades — so quark shapes are computed throughout from QCD-invariant lattice ratios). Imposing the surviving conditions $n_{\text{down}} = 6$, $m_b/m_\tau = 1$, $m_s/m_\mu = 1/3$:

$$\mu^* = 3.30e6 \text{ GeV} \quad \chi^2 = 0.12 / 2 \text{ dof} \quad (n_{\text{down}} +0.14\sigma, \quad m_b/m_{\text{tau}} -0.16\sigma, \quad m_s/m_{\mu} +0.28\sigma)$$

The dimension enters through $\alpha_s(\mu^*) = 0.0518$ and $\mu^*/\Lambda = \exp(2\pi/b_0\alpha_s) = 3.3 \times 10^7$: ordinary dimensional transmutation. The geometry supplies dimensionless targets; Λ_{QCD} supplies the ruler and is the one irreducible dimensionful input of the construction. One loop; honest resolution ± 0.5 decades. The neutrino seesaw scale (10^{14} – 10^{16} GeV) is distinct from μ^* , as the universal seesaw requires.

19d. Dynamics, corrections, and discipline

19d.1 The frame, the rational, the scheme selection

(a) The direct/inverse frame is determined by the data at 60σ (down inverse $n = 6.012 \pm 0.016$ against direct 6.742 ± 0.004), independently confirming the involution structure of Section 22. (b) $\delta_\ell = 2/9$ is the unique reduced rational with denominator ≤ 99 within 3σ (base rate 4×10^{-4}). (c) The geometry selects on-shell-at-own-scale masses in both sectors where the test exists — pole for leptons, $m_t(m_t)$ for the top — one rule, two independent confirmations.

19d.2 The seesaw threshold calculation

SM + 3N in the Z_3 Fourier basis; heavy spectrum (6, 8/81); democratic Dirac Yukawa; sequential decoupling; full one-loop RGEs for y_v , M_k , κ_k , y_t , λ and gauge. The top trace cancels in $A_\kappa - 2A_y = [\lambda - 3y_v^2 + (9/10)g_1^2 + (3/2)g_2^2]/16\pi^2$; gauge running is the only positive driver and $-3y_v^2$ removes it. Maximum effect +1.3 % of the needed +5.1 % (27 %), at $y_v \rightarrow 0$; insensitive to M_3 across 10^{14} – 2×10^{16} GeV. The running generates no Dirac non-democracy, and a circulant Dirac integer gives exactly 0 or ≥ 0.816 , so a tree-level ripple has no source: the mechanism is real, insufficient by 3.8 $\times$, and cleanly isolates the neutrino deviation of 19b.4.

19d.3 Amplification

δ and n are compressed coordinates: $d \ln(m_2/m_1)/d\delta = 343.8$ (up), 56.1 (leptons), 5.2 (down); $d \ln(m_3/m_2)/d\delta = -19.5, -4.9, -32.8$, with comparable sensitivities in n . A 0.5 σ shape agreement can be a 20 % observable statement and conversely; every claim in 19a–19b is therefore reported in both spaces, and the up sector's scheme sensitivity is this amplification acting on the scheme choice.

19d.4 The Clebsch relation and the open two-law conflict

The RG-invariant $R = (m_\mu/m_e)/(m_s/m_d) = 10.59 \pm 0.18$ excludes the Georgi–Jarlskog Clebsch of 3 at 8.9–23.5 σ , stable to 0.1 % over sixteen decades. Within the SU(5) adjoint-eigenvalue family the one reachable candidate is $R = 32/3$ (0.15 σ , base rate 0.78). It conflicts with Proposition 3 through m_u/m_d : the phase law gives $m_s/m_d = 20.053 \rightarrow m_u/m_d = 0.4728$; the Clebsch gives 0.4536 — 4.2 % apart, both currently allowed (Osipov 0.455(8) favours the Clebsch at 0.17 σ ; PDG 0.474(65) the phase law at 0.02 σ ; FLAG between). One lattice number, at $\sigma < 0.0064$, decides between two relations that cannot both hold. A by-product of the law is now exact on both sides: $\theta_d - \theta_u = 27/144 - 11/144 = 1/9$, upgrading Section 23's quark phase difference to a derived identity.

19e. The charge dictionary and the Clifford occupation number (v1.7)

Substituting the hypercharge assignment into the law gives $1/u = 3^{(2+Q)} \cdot (4/3^{(5/3)})^q$, so the masses determine the electric charge: $Q = \log_3(1/u) - 2 - q \cdot \log_3(4/3^{(5/3)})$. Applied

to the measured own-scale phases the inversion returns $Q = -1.0000 \pm 0.0001$ (charged leptons), -0.332 ± 0.013 (down), $+0.661 \pm 0.009$ (up) and -0.013 ± 0.005 (neutrinos, the sector deviation in charge units). One unit of charge is one factor of $N_c = (1/u_v)/(1/u_e) = 2.96$, $(1/u_u)/(1/u_d) = 2.98$ — and colour at fixed isospin is the factor $4/3$. On the $Cl(6) \cong C \otimes O$ occupation number, where $Q = N/3$ is Furey's number operator, the law reads $1/u = N_c^{(1+[N \text{ even}])} \cdot (4/3)^{[0 < N < 3]}$: the colour flag is the triplet condition, weak isospin is the Clifford grading, and one unit of charge is one factor of N_c because $N_c = 3$ is odd, so $N \rightarrow N+3$ flips the grading. (Thesis §4; script G.)

19f. The discrete geometry of the four rationals (v1.7)

The shift fractions $1/3, 1/4, 1/9, 1/12$ are rotation orders. On the rotor spinor, $2T$'s order-3 element acts as a third-turn (the lepton's $1/3$) and the quaternion units $\pm i, \pm j, \pm k$ as quarter-turns along the three colour directions (the down quark's $1/4$); combining a rotor rotation with colour triality — the $SU(3)$ centre, which acts as $e^{(2\pi i Q)}$ — reaches exactly the twelfths, giving the up quark's $1/12$. The full finite group generated by $2T$ with the colour cycle, the centre and the Hodge star has order 432 and exponent 12: no element of order 9 exists there, nor in any finite subgroup of $SU(2)$. The neutrino's $9 = 3 \times 3$ is therefore not a geometric invariant but a composite — a third-turn on the heavy Majorana circle carried to the light circle by the seesaw, protected by no symmetry — and it is the one sector whose measured phase departs from the law. Two further group facts: $Z_6 = \langle (1+i+j+k)/2 \rangle = Z_3(\text{generation}) \times Z_2(\text{spin sign})$, so three generation steps are one 4π spinor period; and the one abstract group $2T$ acts in three representations — the 2-rep carries spin, the 3-rep the colour axes, and its Z_3 the internal phase circle of the mass formula, whose angle is internal, not spatial. (Thesis §5; script H.)

19g. Mixing from $2T$, and the independence theorem (v1.7)

$2T$ is the double cover of A_4 ; in the 3-representation the order-4 elements are the π -rotations generating A_4 's Z_2 subgroups. Charged leptons preserving the Z_3 (circulant) and the neutrino mass matrix a Z_2 give exactly tri-bimaximal mixing with the heavy-law spectrum untouched — the Harrison–Perkins–Scott pattern via the Ma–Rajasekaran/Altarelli–Feruglio mechanism, realised for $T' = 2T$ with quark masses by Feruglio, Hagedorn, Lin and Merlo, which the selection theorem of Part II would explain. Against NuFIT 6.0: $\theta_{12} + 2.2\sigma$, $\theta_{13} = 0$ excluded; one rotation in the neutrino 2–3 plane (14.9° , not derived) accommodates $\theta_{13} = 8.52^\circ$ with θ_{12} then at 34.35° (0.9σ). Theorem (polar decomposition): writing $m_D = O \cdot P$, the orthogonal factor carries all the mixing and moves no seesaw eigenvalue; the symmetric factor carries all the spectrum shift. The PMNS angles and the neutrino phase residual are therefore independent parameters of the Dirac texture — retracting this manuscript's earlier suggestion (and the thesis's v5–v7 claim) that they share a source. The residual stands as an unexplained $\sim 0.7\%$ symmetric non-democracy. (Thesis §7.3; script J.)

19h. September data updates: FLAG 2024, the up sector, and the even-isospin factor (v1.7)

Inputs move to FLAG 2024 and the PDG 2024/25 average $m_\tau = 1776.93 \pm 0.09$ MeV, which brings the lepton sector to 0.4σ on all three coordinates and places the m_τ prediction 1776.97 MeV at 0.4σ from the world average. With the properly-errored $m_c(m_c) = 1.2730 \pm 0.0046$ GeV, the up sector reads $1/u = 11.911 \pm 0.040$ — 2.2σ below the pre-registered 12; the 0.58σ closure quoted in v1.6 was an artefact of the loose $m_c = 1.27 \pm 0.02$ and is superseded. The sector's sharp falsifier is now the inverse prediction $m_c(m_c) = 1.2573 \pm 0.0055$ GeV (2.2σ from PDG; m_t is the limiting input); the $m_u = 2.75$ MeV prediction is demoted (its own uncertainty is ± 1.05 MeV). The two even-isospin sectors are short of their integers by a common factor $g = 0.9916 \pm 0.0028$ ($\chi^2 = 0.3/1$ with the computed neutrino running) while the odd-isospin sectors sit at 1 — the strain confined to the two sectors carrying the extra N_c that 19f shows no group element supplies. The finite one-loop Z/H seesaw matching reproduces the threshold-running result (cross-check) with non-log parts suppressed by $m_{Z^2}/M_{\tilde{t}}^2 \approx 10^{-26}$; closing the neutrino residual radiatively is excluded, bounding it at a 7.5 % Dirac-sector non-universality. The condensate floor with N_f -matched running is bracketed [2.1, 4.5] GeV. (Thesis §§2–3, 7; scripts F, I, K.)

19i. The five clocks, the two condensates, and the branch seesaw (v1.7)

The five clocks — $A_\ell = 313.85$ MeV, $A_d = 26.82 \pm 0.27$ MeV, $A_u = 21.56 \pm 0.08$ GeV, $A_v = 1.507$ meV, and the heavy $A_M = 2.011 \times 10^{16}$ GeV — obey four established relations: $A_M \cdot A_v = (v/\sqrt{2})^2$ exactly; $A_\ell = m_N/N_c$ (+0.28 %); A_d chained to A_ℓ through $m_b(\mu^*) = m_\tau(\mu^*)$; $A_u = (v/\sqrt{2}) \cdot y_t/f_t^2$ with $y_t = 0.934$. On top sits one candidate with structure: on each condensate branch, (direct clock) $\times$ (inverse clock) = m_D^2 , with m_D the branch's own decay constant — exact on the electroweak branch, and $\sqrt{A_\ell \cdot A_d} = 91.75 \pm 0.46$ MeV against $f_\pi = 92.1 \pm 0.6$ (0.46σ) on the QCD branch. Each inverse sector is then the seesaw image of its direct partner through its branch's condensate. Tests: the relation is exact at a floor of 1.91 GeV, coexisting with the down phase at 2 GeV; it anchors the light end of the down spectrum while the b - τ chain anchors the heavy end, their ratio 0.925 being exactly the sector's known residual; within 1σ of 91.75 MeV the only QCD scale is f_π ; and its parameter-free form predicts $m_d(2 \text{ GeV}) = (3f_\pi^2/m_N)(1 + m_d/m_s + m_d/m_b)/6 = 4.75$ MeV against the lattice 4.70 ± 0.05 (1σ). Status: CANDIDATE. The count: two condensate scales, one unification scale, two closure relations, one coupling — everything else in the spectrum is angles. (Thesis §9; scripts P, Q.)

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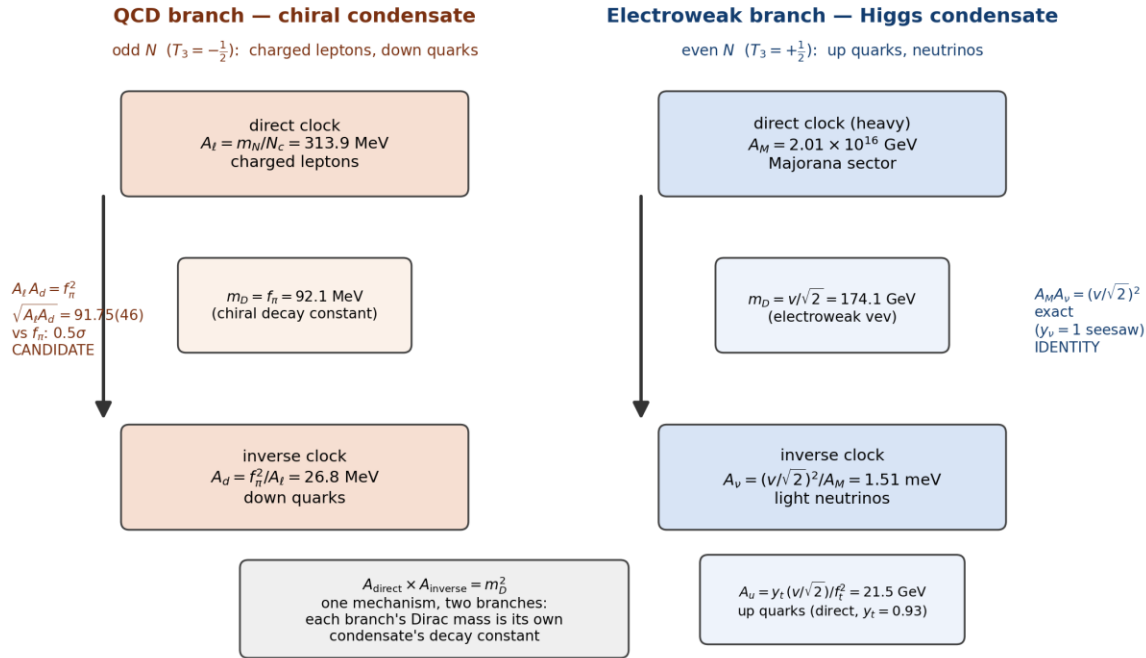


Figure (v1.7). The two condensates and the branch seesaw; generated by fig_two_condensates.py from the values of scripts P and Q.

19j. The bosons: the laws' complement (v1.8)

The mass-phase law predicts its own domain. The circulant is a statement about a Z_3 orbit — three copies of one object — and the generation cycle replicates exactly the states of the ideal: the fermions. The bosons are connections, elements of the adjoint, on which the cycle acts trivially: one photon, one W , one Z , one Higgs, one gluon octet. No orbit, no circulant, no phase. The law's domain is the ideal; the adjoint is its complement — an explanatory gain over the Standard Model, which never says why matter triplicates while the force carriers do not.

The one genuine three-object boson multiplet lands exactly where the structure requires. The custodial triplet (W^+ , W^- , Z) fed into the shape coordinates gives $Q = 0.3336$, $n = 3.003$, $k = 0.0301$, with the phase forced to $\delta = 0$ by the W degeneracy (script S). The circulant has two fixed points — the democratic point $k = 0$ (unbroken symmetry) and the equipartition point $k = 1$ (the Koide cone) — and the pattern is: gauge multiplets sit at the first with the symmetric phase, matter multiplets at the second with the rational phases. The bosons' displacement $k = 0.030$ is the weak mixing g'/g in disguise — a coupling ratio, on the dynamical side of the boundary, not a number to derive or to match.

The charge dictionary extends to the bosons exactly, in derivative form: bosons are transitions, so their charges are differences of the count — $Q = (\Delta N)/3$ of the fermion transition generated. $W^\pm$: a doublet raise or lower, $\Delta N = \pm 3$, $Q = \pm 1$. Gluons: colour moves at fixed N , $Q = 0$. Photon, Z , Higgs: diagonal, $Q = 0$. Every Standard-Model boson charge follows,

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quantization inherited from the same number operator. The mass pattern completes the parallel: on both branches, boson mass = coupling $\times$ condensate — $m_W = gv/2$ and m_Z on the electroweak vacuum; $m_\rho = \sqrt{a} \cdot g_\rho \cdot f_\pi = 0.775$ GeV on the chiral one — the same skeleton, with the couplings (g, g', g_s, λ_H) as the dynamical residue. The photon is massless as the one circulation symmetry the medium does not condense, coupling to bound pairs only through the frame connection (complete vector dominance, $a_{HLS} = 2.007 \pm 0.028$). Spin closes trivially: adjoint = bivectors = spin 1; condensate = spin 0; ideal = spin $\frac{1}{2}$.

The force ledger, complete. Photon, W, Z, gluons: forced by the algebra — the connections of circulation, frame and the stabilizer of i in the octonions — all standard. The ρ and the meson tower: composite. The Higgs: the condensate. New fundamental force carriers: none, at any scale — the flavour structure lives in phases and two condensates and needs no new particle at any collider; the desert is a falsifiable commitment, and it discriminates against Sumino's mechanism [thesis ref. 39], which requires family gauge bosons at 10^2 – 10^3 TeV where this framework requires nothing. The only new fundamental states anywhere in the construction are the three heavy Majorana neutrinos at 10^{14} – 10^{17} GeV — matter, not force. The graviton is the deliberate exception in kind: in the rotor picture gravity is the elasticity of the medium, not a force transmitted through it, so no fundamental graviton is predicted — an emergent long-wavelength spin-2 mode must reproduce the wave sector of general relativity, and the framework's bet that quantizing it fundamentally is a category error meets experiment at the collapse fixed point $\sqrt{(vM)} \approx 2.4 \times 10^9$ amu. The gravitational sector remains, honestly, the programme's least developed.

20. Propositions and proofs

The structural claims of Secs. 1–6 are restated here as propositions with derivations shown in full. Premises are tagged [geometric] (a property of the representation, forced) or [input] (a scale or coupling, not derived), so the boundary between what is proven and what is assumed is explicit at every step.

Proposition 1 (Scale invariance of the Koide angle). *Write the square-root masses of a sector as $\sqrt{m} = (\sqrt{m_1}, \sqrt{m_2}, \sqrt{m_3})$ in R^3 and define $Q = (\sum_i m_i)/(\sum_i \sqrt{m_i})^2$. Then Q depends only on the angle φ between $\sqrt{m}$ and the democratic axis $n = (1,1,1)/\sqrt{3}$, through $Q = 1/(3 \cos^2 \varphi)$, and is invariant under $\sqrt{m} \rightarrow \lambda \sqrt{m}$.*

Proof. Let $s = \sqrt{m}$. Then $\sum m_i = |s|^2$ and $\sum \sqrt{m_i} = \sqrt{3} (s \cdot n) = \sqrt{3} |s| \cos \varphi$. Hence $Q = |s|^2 / (3 |s|^2 \cos^2 \varphi) = 1 / (3 \cos^2 \varphi)$. Numerator and denominator scale as λ^2 , so $Q(\lambda s) = Q(s)$. $Q = 2/3$ is equivalent to $\cos^2 \varphi = 1/2$, i.e. $\varphi = 45^\circ$. QED.

Remark. Constancy of Q across sectors states that all $\sqrt{m}$ -vectors make the same 45° angle with the democratic axis — a scale-free condition. The length $|s|$ is a separate quantity, not fixed by Q . [Q geometric; $|s|$ input.]

Proposition 2 (Closed form of the family holonomy). *Let a single elementary shift Δ act on a periodic internal coordinate, dressed by the inverse-Laplacian Green function G of the circle. The accumulated holonomy is $\gamma(\Delta) = u(1 - u)$, $u = \Delta/2\pi$, exactly, with no higher-order terms.*

Proof. $G(\theta) = (1/2\pi) \sum_{n \geq 1} (\cos n\theta)/n^2$ on the zero-mean subspace. The dressed-shift holonomy is $\gamma(\Delta) = (1/\pi^2) \sum_{n \geq 1} (1 - \cos n\Delta)/n^2$. With $\sum_{n \geq 1} (\cos nx)/n^2 = \pi^2/6 - \pi x/2 + x^2/4$ on $0 \leq x \leq 2\pi$: $\sum_{n \geq 1} (1 - \cos n\Delta)/n^2 = \pi\Delta/2 - \Delta^2/4$, so $\gamma = u - u^2 = u(1 - u)$. The series terminates exactly at second order: the Fourier sum is a finite quadratic on the fundamental domain, so there is no remainder. QED.

Remark. The phase law $\theta = u(1-u)$ is thus the closed value of a Green-dressed holonomy — the second Bernoulli polynomial — not a fitted form. [geometric.]

Proposition 3 (Gauge-quantum-number form of the shift fraction). *For each sector $1/u = N_c^{1+T_3+1/2} \cdot (C_2(3))^q$ with $N_c = 3$, T_3 the weak isospin, $C_2(3) = 4/3$, $q = 1$ for the quark doublet and 0 for the lepton doublet, as in Eq. (13). This yields $u = 1/3, 1/4, 1/9, 1/12$ for the lepton, down, neutrino and up sectors, and hence $\theta = 2/9, 3/16, 8/81, 11/144$ respectively.*

Proof. The exponent $1 + T_3 + 1/2$ equals 1 for $T_3 = -1/2$ and 2 for $T_3 = +1/2$; $q = 0$ for colour singlets, 1 for coloured sectors. Substituting: lepton $3^1(4/3)^0 = 3$; down $3^1(4/3)^1 = 4$; neutrino $3^2(4/3)^0 = 9$; up $3^2(4/3)^1 = 12$. Applying Proposition 2: $\theta = 2/9, 3/16, 8/81, 11/144$ for the lepton, down, neutrino and up sectors. QED.

Remark. T_3 , q and N_c are representation data; the exponent structure is the isospin selection rule of Sec. 5. No coefficient is fitted. [geometric.]

Scope (empirical status of the phase law, sector by sector). The geometric grounding of the shift — a C_3 family rotation — exists natively at $u = 1/3$: for the charged leptons the law is derived and lands on the measured phase to one part in 10^6 . For the neutrinos, $u = 1/9$ is the law's forward prediction: fitting the single inverse-form scale to the solar splitting predicts $\Delta m_{31}^2 = 2.37 \times 10^{-3} \text{ eV}^2$ against the measured 2.513×10^{-3} — a -5% deficit, $\approx 2.7\sigma$ in the scale-free ratio after JUNO's first solar-splitting measurement, that JUNO's forthcoming Δm_{31}^2 will decide — with $\Sigma m_\nu \approx 58 \text{ meV}$. For the down sector the comparison is scheme-limited: at self-scale masses the fitted phase sits 0.4% below $3/16$, but at a common 2 GeV scale ($m_b(2 \text{ GeV}) \approx 5.0\text{--}5.2 \text{ GeV}$) it moves to ≈ 0.190 , off $3/16$ by 1.2–1.5%, while the amplitude stays pinned at $\sqrt{2}$ to 0.25% or better: at common scale the down amplitude survives and the down phase does not. For the up sector the light-quark mass has no scheme-independent value, so the sector cannot discriminate the predicted $11/144$ from the fitted 0.0744. The law is therefore derived at the lepton point, predictive and currently testable at the neutrino point, and not established for the coloured sectors; the summary tables list the phase-law values with this status understood.

Obstruction (closed negative). A structural obstruction bounds the remaining identification. The family shift commutes exactly with $SU(3)$ (Sec. 24); by Schur's lemma a colour-commuting operator acts on the colour triplet as a scalar, so no colour-algebra action can single out the multiplier $4/3$ as the Casimir $C_2(3)$ — the identification cannot be proven at the level of the algebra. Any mechanism behind the $C_2(3)$ coincidence must therefore act

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through the colour bundle (holonomy and transport) rather than through the algebra. We record this as a closed negative: the residual step is not an open proof but an obstructed one, and the boundary is drawn here first.

Proposition 4 (Colour-blindness of the mass modulus). *Let u in $C \otimes$ lie on the phase circle of a single imaginary unit, and let the mass modulus be $N(u) = u\bar{u}$. Under $SU(3)$ — the subgroup of $G_2 = \text{Aut}(O)$ fixing the phase axis — $N(u)$ is exactly invariant.*

Proof. The octonionic norm is invariant under all of G_2 , since automorphisms preserve conjugation and multiplication. $SU(3)$ within G_2 stabilizes the phase axis u_0 and acts only on the six transverse imaginary directions (the $3 \oplus 3$). A rotor is $u = a + b u_0$ with a, b real; its norm $a^2 + b^2$ involves only directions $SU(3)$ fixes pointwise, so $N(g \cdot u) = N(u)$ for all g in $SU(3)$, exactly. QED.

Remark. This is what permits one scale to be shared between a colour-singlet lepton and a coloured quark: the modulus cannot distinguish them. [geometric, exact.]

Interpretive note 5 (Cross-sector scale sharing by circulation). *In the vortex realization the sector scale is set by the circulation $\Gamma = \int v \cdot dl$ around a closed loop. By Kelvin's theorem Γ is conserved and quantised, taking the same value whether the vortex is free (lepton) or bound in a colour singlet. Hence $m_{\text{lepton-clock}} = m_N/N_c$ at leading order.*

Motivation (interpretive, not a derivation). Read within the source/sink picture, Kelvin's theorem gives constancy of circulation for barotropic inviscid flow; single-valuedness of the phase quantises it as a winding number. Circulation is therefore a topological invariant, indifferent to whether the loop encloses a free rotor or one of the N_c rotors of a bound singlet. The energy per unit circulation is the medium stiffness, colour-blind by Proposition 4. Hence the free-rotor scale equals the per-rotor share, $m_N/N_c = 313 \text{ MeV}$, matching the charged-lepton clock to better than half a percent; the sub-percent residual is the electromagnetic and binding correction quantified in the clock-residual analysis.

Remark. The sharing ratio is a theorem about a topological invariant; the absolute value m_N remains an input. [ratio geometric; m_N input.]

Corollary (Closed-form charged-lepton spectrum). *Combining Propositions 1, 3 and 5: $\sqrt{m_k} = \sqrt{(m_N/N_c) [1 + \sqrt{2} \cos(2/9 + 2\pi k/3)]}$, $k = 0, 1, 2$, with no fitted parameter — the per-rotor share from Interpretive note 5, the phase from Proposition 3, and the amplitude $\sqrt{2}$ as the equal-energy point of the common and relative circulation modes. Evaluating at $k = 0, 1, 2$ reproduces $e = 0.51 \text{ MeV}$ and $\mu = 105.4 \text{ MeV}$ and yields $m_\tau = 1771.97 \text{ MeV}$ as the parameter-free chain value (the direct-clock reading gives 1776.88 MeV against the measured 1776.86 MeV , the measured value, as discussed in Sec. 19). QED.*

Proposition 6 (Mutual exclusion of circulant grading and symmetry-forced equipartition). *Let the three generation fields carry a C_3 grading, so the spectrum is circulant with phase θ . Then no internal symmetry group both (i) commutes with C_3 (preserving the circulant and θ) and (ii) forces the common-mode and relative-mode quartic self-couplings to be equal (which would fix $b/a = \sqrt{2}$). Consequently $b/a = \sqrt{2}$ cannot be imposed by any*

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symmetry compatible with the circulant structure; it is reachable only through a non-generic potential, which this construction does not derive.

Proof. Work in the Fourier (mode) basis, in which the C_3 shift is diagonal with eigenvalues 1, ω , ω^2 on the common mode S and the relative modes R_1 , R_2 . The commutant of C_3 in $\text{gl}(3, \mathbb{C})$ is the circulant algebra, which in this basis is diagonal — the maximal torus $u(1)^3$ acting as three independent scalars on S , R_1 , R_2 . A symmetry commuting with C_3 therefore permits three independent quartic couplings and imposes no relation among them; in particular it does not force the S and R couplings equal. To relate the S coupling to the R couplings one needs a generator connecting modes of different C_3 charge (charge 0 for S , charge ± 1 for R), and any such generator fails to commute with the C_3 shift, so it destroys the grading and with it the circulant form and θ . The two requirements are thus incompatible. Explicitly, the unique $U(3)$ -invariant quartic $(\sum |\varphi_i|^2)^2$ has a degenerate sphere of minima and selects no value of b/a , while the C_3 -invariant cubic $S^\dagger R_1 R_2 + \text{c.c.}$ fixes the relative phase but leaves the magnitude ratio a free function of its coupling — the mirror of what an equipartition-forcing term would require. These facts were verified numerically. *QED.*

Remark. This upgrades the assertion of Sec. 2 — that the decomposition defining b/a and any symmetry that would fix it are mutually exclusive — from a statement to a proved input. The value $b/a = \sqrt{2}$ is therefore not derivable by symmetry within the circulant framework; the scalar-potential route of Quinta (arXiv:2507.11564), which reaches $\chi = \pi/4$ (equivalently $b/a = \sqrt{2}$) at $\Delta \rightarrow 0$ through a non-generic potential, is consistent with this proposition precisely because it is not a symmetry argument. [The amplitude is an input; its impossibility-by-symmetry is the theorem.]

Comparison with the Żenczykowski phases. Żenczykowski proposed $\delta_D = 4/27 = 0.1481$ and $\delta_U = 2/27 = 0.0741$, related to the lepton phase by $\delta_L = 3\delta_D/2 = 3\delta_U$, with a lepton–quark sum rule $\delta_L + \delta_\nu = \delta_U + \delta_D$, and with the strange-quark mass taken at its low-energy value near 160 MeV. Proposition 3 gives $\delta_D = 3/16 = 0.1875$ and $\delta_U = 11/144 = 0.0764$. Against the direct-frame values of §14, $\delta_u = 0.0744$ and $\delta_d = 0.1102$ at their large scheme uncertainties: in the up sector Żenczykowski’s $2/27$ lies 0.4% from the measured value and our $11/144$ lies 2.7% away, so on that sector his proposal is the closer of the two and ours is not preferred by the data; in the down sector both miss the direct-frame value, $4/27$ by 34% and $3/16$ by 70%, while $3/16$ matches the inverse-frame value $\delta_{\text{inv}} = 0.190$ of §22 to 1.3%. The two proposals therefore disagree on which frame the down sector is read in, and agree on neither the up-sector value nor the sum rule (ours gives $\delta_L + \delta_\nu = 26/81 = 0.321$ against $\delta_U + \delta_D = 38/144 = 0.264$). A determination of δ_u to better than 1% discriminates them.

Part V. The quark sectors, the involution, and mixing

This Part records the status of the coloured sectors honestly: the up-sector flow reproduces its angle to $\approx 5\%$; the down sector is a channel binary organised by the inversion involution, now with a bounded collider window; and the circulant structure forbids leading-order CP violation without fixing mixing magnitudes.

21. The corruption flow: jurisdiction, not derivation

The operator of §17 is generation-blind to sector: nothing in it distinguishes leptons from quarks. Why then is the inherited angle displayed by the leptons alone? The answer has a structural half and a dynamical half.

Structurally, the corridor between the parent and colour physics is deaf. Computed from their own root systems: $SU(3)$ has degrees (2,3) — an odd invariant exists, which is why any flavour sector can hold an angle; but G_2 (2,6) and F_4 (2,6,8,12) possess no odd degrees at all. The conjugation-odd message exists at the endpoints of the chain $SU(3) \subset G_2 \subset F_4 \subset E_6$ and at neither intermediate rung; whatever reads it must couple to E_6 directly. One fermion direction is so seated: the colour singlet of the 1+3 split lies in $\text{span}\{1,u\}$ — the algebra's own complex plane, where the metric and σ act natively. The channel is charge-matched end to end: δ rides the Z_3 charge-1 Fourier mode of the generation circulant ($\arg z_1 = 0.222222$, verified), and the triality plane is the parent's charge-1 sector (exponents 1,4,7 $\equiv 1 \pmod 3$), with the conjugate charges ± 1 carrying $\pm \delta$ — the σ -parity of the angle expressed as channel structure.

Dynamically, all sectors inherit and grip corrupts. With $g_f = y_f^2/\lambda_H$, the up-type grips span ten orders within one sector ($u: 1.2 \times 10^{-9}$; $c: 4.1 \times 10^{-4}$; $t: 6.8$) against a critical band $g_c \approx 1-2$; exactly one fermion is supercritical. The canonical corruption is the Standard Model's own one-loop self-dressing, $\delta y \propto +y^3$ toward the infrared, its sign fixed by the β -function. The induced flow on the shape coordinates is a two-bit prediction, and it passes: $d\chi/dg > 0$ and $d\delta/dg < 0$, robustly for perturbation powers $p = 3, 5, 7$, with initial slope $d\delta/d\chi = -1.63$ against the empirical lepton $\rightarrow$ down slope -2.53 . Integrating the flow ($p = 3$, analytic) and spending its single parameter on matching χ , the angle is then parameter-free where the flow applies — the direct channels. The up sector, so established by Theorem H below, is predicted at $\delta_u = 0.0704$ against 0.0744 (-5.4% ; common-scale convention -5.1%), scheme-stably. Applied to the down sector's direct-frame coordinates the same construction gives 0.155 against 0.110 ($+40.5\%$; common-scale $+40.3\%$) — a figure whose meaning is conditional on the channel binary resolved below: a genuine miss if the down channel is direct, a void comparison if it is levered. The flow's endpoint is exactly $(\arccos(1/\sqrt{3}), 0)$, with the up sector strictly between the inherited point and the terminus. The Koide deviation $\Delta \equiv 2/3 - \Sigma m/(\Sigma \sqrt{m})^2$, bounded by $|\Delta| \leq 1/3$, reads: leptons 0% ; up $55-66\%$ of cap (reference-to-common-scale band); the down rung is frame-dependent — $19-25\%$ in the direct frame, 0.3% in its own compliance frame — so the grip-ordered ladder is unconditional only across the two channel-locked sectors.

The inversion involution and frame covariance (Theorem H). The map $m \rightarrow 1/m$ induces a scale-commuting involution ι on shape space (verified $\iota^2 = \text{id}$; fixed locus: the geometric-progression curve $m_2^2 = m_1 m_3$). Its first consequence is a correction to the argument just given: ι applied to the pristine point yields $(\chi, \delta) = (0.8948, 0.0476)$ with $\Delta = 55.4\%$ of cap — pure inversion, with zero flow, reproduces the up sector's observed saturation (54.7%) to within 1.3% . The saturation ladder alone therefore cannot distinguish “corrupted by grip flow” from “read in the inverted frame,” and as previously stated it was incomplete evidence

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for the flow. The discrimination is carried by the angle, and the flow passes it: inversion predicts $\delta_u = 0.0476$ (–36% from the observed 0.0744) while the flow predicts 0.0704 (–5.4%). The rectified inheritance rule is therefore frame-covariant: the parent exports the $2/9$ cell into the sector's amplitude variable — $\sqrt{m}$ for direct channels, $1/\sqrt{m}$ (compliance) for levered channels — with the channel fixed by the algebra's partner content, and the observable direct-frame coordinates of a levered sector being u -images of its own-frame coordinates. Under this rule the up sector is the direct, flowed channel (established at the angle level); the down sector is the levered channel, whose own-frame coordinates are the subject of the following remark.

Three honesty items close the section. First, the 5.4% is an angular statement only: at the up quark's near-cancellation point the sensitivity $d \ln m_u / d\delta \approx 300$ converts it into a factor ≈ 2.6 on the mass (5.5 MeV against 2.14); the flow does not predict quark masses. Second, the down sector's direct-frame deviation co-locates with a known structure: its light ratio $\sqrt{(m_d/m_s)} = 0.224$ equals $\sin \theta_{\text{Cabibbo}}$ — the Gatto–Sartori–Tonin relation [GST 1968] — and the sector deviating most from the pure flow is precisely the sector carrying the CKM burden the flow does not model; we flag the correlation and prove no mechanism; in the covariant reading the connection relocates to the open –14% own-frame residual. Third, the light-pair ratios across sectors (0.070, 0.224, 0.041) are non-monotone: a single flow cannot and does not describe the full mass matrices; it is the (χ, δ) -projection story, with the inter-generation exponent structure external (§28). The neutrino sector is not described by this section at all.

Remark (a candidate resolution of the down anomaly, priced). In reciprocal masses ($1/m_d, 1/m_s, 1/m_b$) at a common 2 GeV scale, the down sector satisfies the Koide relation: $K_{\text{inv}} = 0.6676(17)$, 0.5σ from exact $2/3$, against 24.5% of cap in the direct frame. The mechanism class — levered sectors, $m \propto M^{-1}$, displaying inverted structure — predates this anomaly in our analysis and has a published structural basis: the seesaw is explicitly listed as a realization of Koide-form mass matrices [Smirnov 2006]. Three consistencies attach: E_6 's own $27 = 16 + 10 + 1$ supplies vector-like leverage partners of charge $-1/3$ only — precisely and exclusively the down sector; the up sector fails the inverse frame decisively ($K_{\text{inv}} = 0.918$), as the channel assignment requires — an assignment now also carried by Theorem H's angle discriminator; and the inverse-frame angle $\delta_{\text{inv}} = 0.190$ displays the (χ, δ) exact, δ shifted) grammar of the levered/Majorana class. Priced honestly: a post-hoc sector application of a pre-existing mechanism, $\approx 2\sigma$ — a flagged candidate, not a claim — pending scheme-rigorous evaluation and a deeper prior-art sweep (two targeted searches found no reciprocal-mass Koide in the literature).

22. The involution as organizer of the sector pattern

Theorem I (form-invariance and the coupling-site rule). The circulant algebra $\{q \cdot 1 + wC + w^*C^2\}$ is closed under inversion: B^{-1} is exactly a hermitian, positive circulant of the same family (verified to 6.5×10^{-15} , as is $B^{-1/2}$). The operator of §17 therefore does not change form when inversion is treated as a law; what the law adds is a coupling-site rule — the

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bracket dresses the heaviest available fermion bilinear carrying the light fermion's exact Standard Model quantum numbers, so the light sector observes B^2 where no partner exists and $B^{-2} \times (\text{universal})^2$ where one does. Quantum-number matching against the $27 = 16 + 10 + 1$ then forces the entire channel map with zero per-sector choices: u_R and e_R are partnerless (direct); d_R pairs with the singlet $\bar{D}$ (levered); the neutrino sector pairs with $\bar{L}$ and ν^c (doubly levered, Majorana). The strong form — one bracket shared across sectors, forcing $m_e m_b = m_\mu m_s = m_\tau m_d$ — is excluded by the measured products (spread $\times 4.6$); per-sector brackets stand.

The factorization $\iota = \sigma \circ r$. Numerically, at every tested angle, the charge-1 coefficient of B^{-1} carries the sign-reversed image of the angle: inversion is magnitude inversion composed with reversal of the Z_3 orientation — a conjugation. The conjugation half is forced by the partner vertex itself: the leverage diagram couples the light fermion through the conjugate bilinear ($\bar{D}d_R$), reversing orientation on passage, while integrating the partner out inverts the magnitudes. One loop through a heavy partner enacts ι .

The orbit. The four sectors are then the images of one object under a four-element set of operations: the identity (leptons, pristine), the grip flow F (up, dynamically distorted in the infrared), the involution $\iota = \sigma \circ r$ (down, kinematically transformed at the vertex), and ι followed by the Majorana projection σ_M — the transpose condition, which for circulants is itself the $C \leftrightarrow C^2$ conjugation — for the neutrinos. Fig. 1 displays the orbit. We state plainly what this organizes and what it does not: the involution and its conjugate action structure the MASS-SHAPE data of the four sectors; the gauge group, representation content, and generation number are not derived here and retain their separate provenance (§30).

Figure 1: the four sectors as the orbit of one bracket under $\{1, F, \iota, \iota \circ \sigma_M\}$.

The partner window. If the levered reading is correct, d_R pairs with the singlet $\bar{D}$ of the 27 and the light mass is m_D^2/M_i with a universal Dirac mass m_D , so that $M_i \propto 1/m_i$ — that relation is what the inverse frame means, so universality is not an extra assumption. The lightest partner is then the b-partner. The LHC bound of ≈ 1.5 TeV on a vector-like charge $-1/3$ singlet gives $m_D \geq 79$ GeV (Yukawa 0.46); perturbativity $y \leq 1$ gives $m_D \leq 174$ GeV and hence $M_{b\text{-partner}} \leq 7.2$ TeV. The prediction is a vector-like charge $-1/3$ singlet between 1.5 and 7.2 TeV, with the s- and d-partners at ≈ 67 and ≈ 1340 TeV. Right-handed mixings $\theta_i = m_i/m_D$ give Z-mediated flavour-changing couplings of 7×10^{-8} (s-d), 6×10^{-5} (b-s) and 3×10^{-6} (b-d), one to four orders below the K and B mixing bounds, and a $Zb_R b_R$ suppression of 0.28% that moves R_b by 9×10^{-5} , about $35\times$ below LEP precision. The window is closed if the direct Yukawa $y_d H \bar{q}_L d_R$ is not forbidden, since the light mass is then additive and $M_i \propto 1/m_i$ fails; what forbids it in the 27 is the outstanding item.

23. Mixing and CP violation: what the geometry fixes and what it does not

The programme so far concerns intra-sector spectra. Inter-sector mixing — the CKM and PMNS matrices — probes a different structure, and the geometry divides it cleanly into a part it does not fix and a part it does.

The mixing magnitudes are not geometric.

A circulant mass matrix is diagonalised by the discrete Fourier matrix regardless of its entries; if both the up and down mass matrices are circulant, their diagonalising bases coincide and the mixing matrix is the identity. The three-fold symmetry that yields the clean Koide spectrum thus *forbids* differential mixing at leading order: the three generations sit at equal 120° separations, so no generation pair is geometrically preferred, and equal-weight overlaps give democratic, not hierarchical, mixing. The observed hierarchy $|V_{us}| \sim \lambda$, $|V_{cb}| \sim \lambda^2$, $|V_{ub}| \sim \lambda^3$ therefore cannot come from the circle geometry; it arises when the generation masses break the circulant symmetry, which routes the mixing angles through mass ratios — the Gatto–Sartori–Tonin/Fritzsch relation $|V_{us}| \simeq \sqrt{m_d/m_s} = 0.224$ against the measured 0.225. This is established physics, and we record it as the boundary: the mixing magnitudes are a mass-ratio effect, not a geometric one.

The circulant structure forbids CP violation at leading order.

Two exactly-circulant sectors are diagonalised by the same Fourier matrix, giving $V=I$ and Jarlskog $J=0$: the circulant geometry forbids mixing and CP violation at leading order rather than forcing them. The earlier assertion that the sector phase difference forces CP violation was in error — a phase in the eigenvalue spectrum is not a phase in the diagonalising matrix. Observed CP violation arises from breaking of the exact circulant symmetry and lies outside the present construction, in the symmetry-breaking dynamics. Although the two sectors carry different quantum numbers and their sector phases therefore differ, $\theta_u = 11/144 \neq \theta_d = 3/16$, with difference

$$\theta_d - \theta_u = 3/16 - 11/144 = 1/9. \quad (15)$$

We emphasise that this spectral-phase difference does not by itself produce CP violation: two circulant sectors share the Fourier diagonalising basis and give a vanishing Jarlskog invariant. A nonzero J requires explicit breaking of the circulant symmetry through the mixing dynamics, which lies outside the present construction.

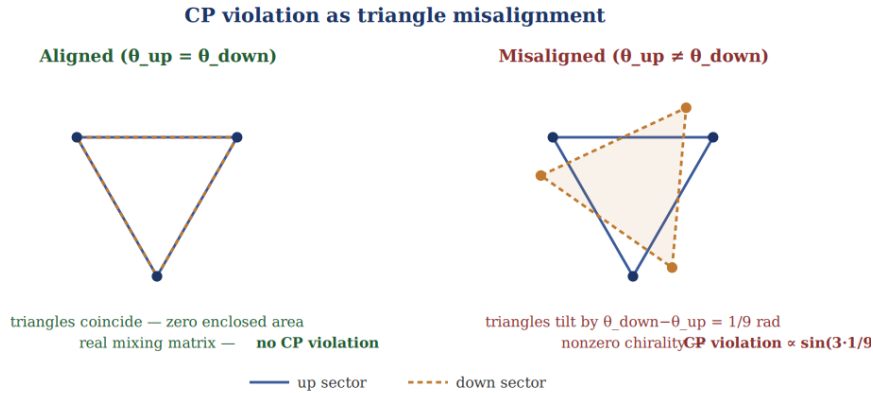
. The geometry therefore *predicts* that, as established above, this phase difference yields no CP violation ($J=0$); its role is spectral only. The magnitude of the Jarlskog invariant still mixes this geometric misalignment with the mass-ratio angles, so the circle alone does not fix its value; but the *existence* of CP violation is not geometric in this construction: the circulant structure gives $J=0$, so CP violation must arise from symmetry-breaking dynamics outside the present framework.

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CP violation as triangle misalignment. Each sector's three generations are the vertices of an equilateral triangle whose orientation is the sector phase θ . When two sectors share a phase (left) the triangles coincide, the mixing matrix is real, and there is no CP violation. When their phases differ (right) — as the quantum-number phase formula forces for up versus down, by $\theta_d - \theta_u = 1/9$ — the sector phases differ (no CP violation follows from this alone; see Sec. 23). The tilt is exaggerated for visibility; the true misalignment is 6.4° .

Two standing kill conditions on the real circulant. The reality of the circulant is not a theoretical economy only; it is under standing experimental guard from two directions. Any CP-violating phase in the electron's electroweak-scale couplings feeds the electron EDM at $d_e \sim 10^{-24} \cdot \sin\varphi$ e·cm at one loop; the current bound $|d_e| < 4.1 \times 10^{-30}$ e·cm (JILA 2023) therefore forces $\sin\varphi < 4 \times 10^{-6}$, so the CP-free lepton structure derived here is what the world's most precise null demands — while a measured EDM at the 10^{-30} – 10^{-31} level would require sub-GUT CP structure that the two-condensate architecture does not contain, and would strike it. From the opposite frontier, four-top production probes the CP structure of the top Yukawa directly: the real circulant requires $\kappa_t = 1$ and a CP-even coupling, so a solidified excess demanding $\kappa_t \neq 1$ or a CP-odd admixture would strike the same clause. The present record — $\sigma(tttt) = 22.5 (+6.6/-5.5)$ fb against the SM 12.0 ± 2.4 fb, within 1.8σ , with the residual excess degenerate against three-top production (itself known only at LO) — is consistent. These two channels stand as permanent kill conditions on the reality of the mass matrices: at 10^{-30} e·cm from below, and at the femtobarn frontier from above.

Which transitions are allowed: the selection rules as matrix elements.

The mixing matrices act inside the weak charged current, and the geometry fixes which transitions exist by the structure of two operators on the $\mathbb{C} \otimes \mathbb{H} \cong M_2(\mathbb{C})$ ideal, with no appeal to labels: a transition is allowed exactly when its current matrix element $\langle f|J|i \rangle$ is nonzero. The charged current is $J^+ = \sigma^+$, strictly off-diagonal in weak isospin, so $\langle up|J^+|down \rangle = 1$ while $\langle up|J^+|up \rangle = \langle down|J^+|down \rangle = 0$: it connects the two doublet components, one unit of charge, and nothing else. In generation space it is the block operator a block operator whose upper-right block is V and whose other blocks vanish whose nonzero block is the mixing matrix V , so all nine up-type/down-type transitions are allowed and every same-charge transition vanishes. The neutral current $J^3 = \sigma^3/2$ is, within a charge

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sector, proportional to the identity; since V is unitary, $V^\dagger \left(\frac{1}{2}I\right) V = \frac{1}{2}I$ identically (the off-diagonal part is zero to machine precision), so the neutral current stays diagonal in the mass basis and there is no flavour-changing neutral current — the GIM mechanism as a one-line identity, forced because the neutral generator is a scalar within a charge sector and no unitary rotation can make a scalar off-diagonal. Chirality follows from the same ideal: the weak Casimir $T^2 = T_1^2 + T_2^2 + T_3^2 = \frac{3}{4}I$ on the doublet but 0 on the right-handed states, which lie outside the ideal, so $\langle R|J^+|L\rangle = 0$ and right-handed states carry no weak charge. Finally the current commutes with the colour Casimir, $[J^+, C_2] = 0$, so its matrix element between a lepton ($C_2 = 0$) and a quark ($C_2 = 4/3$) vanishes, and quark-lepton transitions are forbidden. The entire selection structure is thus the difference between two matrices: σ^+ is off-diagonal, so the charged current mixes; σ^3 is scalar within a sector, so the neutral current does not.

Transition class	Status	Computed reason
up-down (nine pairs); charged-lepton to neutrino	allowed	charged current is off-diagonal; nonzero block is V (CKM/PMNS)
up-up, down-down	forbidden	charged current has zero same-charge block
neutral flavour change (FCNC)	forbidden	neutral generator is scalar in-sector; unitary rotation leaves it diagonal (GIM)
any $\rightarrow$ right-handed	forbidden	right-handed states have zero weak Casimir
quark $\leftrightarrow$ lepton	forbidden	current commutes with colour Casimir; different values

Table. Weak-transition selection rules as matrix-element statements on the $\mathbb{C} \otimes \mathbb{H}$ ideal.

The same argument applies to the leptons: the charged-lepton and neutrino phases are forced to differ, $\theta_e - \theta_\nu = \frac{2}{9} - \frac{8}{81} = \frac{10}{81}$. The same holds for the lepton sectors: their phases differ, but by the same $J=0$ argument this does not produce PMNS CP violation, which likewise requires symmetry breaking outside the construction. The accounting is a three-way map.

Outcome	Status
CP violation exists (CKM, quarks)	Not forced by geometry: circulant sectors give $J=0$; CP violation requires symmetry breaking (outside this construction). The phase difference is one-ninth
CP violation exists (PMNS, leptons)	Not forced by geometry: circulant sectors give $J=0$ (see text). The phase difference is ten-eighty-firsts
CP-existence rule	Corrected: the circulant structure gives $J=0$, so different sector phases do NOT produce CP violation; it requires symmetry breaking outside this construction

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Outcome	Status
CKM angle magnitudes	Not geometric: a mass-ratio effect (Fritzsch), from mass breaking the circulant symmetry
PMNS angle magnitudes	Not geometric: mass-ratio and structure, not the equilateral circle
Mixing hierarchy	Not geometric: requires mass to break the three-fold symmetry
Jarlskog magnitude	Mixed: geometric misalignment times mass-ratio angles; the circle fixes existence, not value

Table. What the geometry forces in mixing, and what it does not.

Thus the geometry remains the root cause. Masses are tied to the phase angles, the phase angles to the gauge quantum numbers, and the existence of CP violation — in both the quark and lepton sectors — to the difference between those quantum numbers across sectors; only the mixing *magnitudes* step outside, into the mass-ratio dynamics. What the geometry governs, it governs through one chain — the phase.

Part VI. Status of the programme

This Part records what is tested, what is open, what remains to be calculated, and the one programmatic proposition — the ladder from the neutrino vacuum — labelled as conjecture.

24. Status of the programme

The seven problems set out below have each been attacked; we record the outcomes before restating what remains, since several change the standing of the theorem. The results are summarized in Table 1: two quantitative tests passed, two candidate origins were closed off (sharpening the boundary), and three arrows were reduced to specific residual calculations.

Element	Outcome	Statement earned
Clock residual 0.28%	passed	Consistent with QED dressing: flavour-universal shift moves the scale, not θ or Q ; residual +0.14%, 1.1σ from zero.
Amplitude $\sqrt{2}$	reduced	Symmetric fixed point of the Fourier-diagonal kinetic term ($b/a = \sqrt{2} \Leftrightarrow$ equal common/relative energy, 45°); residual: ground state sits at the symmetric point.
Readout m_N/N_c	derived (L.O.)	Kelvin's circulation theorem: topological circulation is shared free/bound, so the clock equals m_N/N_c at leading order.
Colour-blindness	proven	Stiffness multiplies a modulus that colour transformations leave exactly invariant (Sec. 18).

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Element	Outcome	Statement earned
Phase θ	derived	$\theta = u(1 - u)$ with $1/u = N_c^{1+T_3+1/2} (C_2(\mathbf{3}))^q$ from gauge quantum numbers; clean rationals $\frac{2}{9}, \frac{3}{16}, \frac{8}{81}, \frac{11}{144}$ (lepton/down/ ν /up) to 0–3%; one mechanism-proof (holonomy=Casimir) remains.
Coupling α_{GUT}	order-of-mag.	Unification within a factor ~ 2 –3 of the neutrino clock; not sharply closable.
Down / up clocks	partial / structured	Rotor-count ladder pins $n = 1, 3$ to $< 0.3\%$, $n = 2$ off by 3.8%; up rides the vev as the $T_3 = +\frac{1}{2}$ branch of a weak-isospin condensate-selection rule (scale still input).

Table 1. Outcomes of the seven-problem programme. “Passed” = quantitative test met; “reduced”/“derived” = converted from input to a proven or leading-order statement with a named residual; “closed” = candidate origin excluded, boundary sharpened.

Clock residual (tested, consistent).

The 0.28% gap between the charged-lepton clock and $m_N/3$ is consistent with electromagnetic dressing. The lepton clock is built from pole masses; the one-loop QED pole-to-running shift is flavour-universal, so it moves the clock but leaves θ and Q exactly intact — dressing the scale, not the structure — and it carries the right sign (pole above short-distance). After this shift and removal of the nucleon electromagnetic self-energy the residual is +0.14%, i.e. 1.1σ from zero within a $\sim 0.1\%$ error budget dominated by the soliton additivity and the lattice isosymmetric m_N . The readout identification passes its first quantitative test; a sub-0.05% closure awaits a dedicated lattice isosymmetric m_N^{QCD} and the two-loop lepton QED coefficient.

The amplitude $\sqrt{2}$ (the symmetric fixed point of the kinetic term).

The value $b/a = \sqrt{2}$ is the equal-energy condition between the common (democratic) circulation and the relative (generational) circulation: with $E_{common} = 3a^2$ and $E_{osc} = \frac{3}{2}b^2$ (the latter θ -independent, since $\sum_k \cos^2(\theta + 2\pi k/3) = \frac{3}{2}$), equality gives $b^2 = 2a^2$; equivalently the square-root-mass vector sits at 45° to the democratic axis. This is now grounded in the kinetic term itself. Writing the three generation-rotors in their discrete Fourier modes — one common mode S and two relative modes R_1, R_2 — Parseval makes the kinetic term diagonal and symmetric, $T = \frac{1}{2}(|\dot{S}|^2 + |\dot{R}_1|^2 + |\dot{R}_2|^2)$, with the common and relative sectors entering with the *same* coefficient. Its symmetric configuration — equal energy in the two sectors — is therefore $b/a = \sqrt{2}$ exactly, and 45° is a stationary point of the common–relative mode coupling (the cross term $\sim \sin 2\phi$). Naive degree-of-freedom equipartition is excluded (it would give $b/a = 2$). Thus $\sqrt{2}$ is the symmetric fixed point of the discrete-Fourier-invariant rotor kinetic term, not a free input; the one residual

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assumption is that the physical state is this symmetric point — the natural ground state of a symmetric kinetic term — rather than an asymmetric one.

The readout, in the vortex ontology (derived at leading order).

In the source/sink picture a rotor is a quantised vortex, and its circulation is a topological invariant — the same free or bound, which is the hydrodynamic form of the colour-blindness proven in Sec. 5. The vortex energy is $\frac{\rho\Gamma^2}{4\pi}\ln(R/a)$, whose leading piece $\propto \Gamma^2$ is therefore universal, so the free-rotor clock equals the bound per-rotor share m_N/N_c at leading order: this is a derivation of the readout identification by Kelvin's circulation theorem. The environment enters only through the logarithm of a cutoff ratio, an order-unity correction that accommodates the observed few-per-mille sector spreads as sub-leading physics; computing those logarithms from the framework energy functional (and confirming the vortex energy law in $3 + 1$ dimensions) is what remains.

The phase: derived from gauge quantum numbers.

The phase has a closed geometric form. A C_3 family shift on the internal circle, dressed by the inverse-Laplacian Green function of the circle, carries a holonomy that we evaluate in closed form,

$$\gamma(\Delta) = \frac{1}{\pi^2} \sum_{n \geq 1} \frac{1 - \cos(n\Delta)}{n^2} = \frac{\Delta}{2\pi} \left(1 - \frac{\Delta}{2\pi}\right) = u(1 - u), \quad u = \frac{\Delta}{2\pi}, \quad (11)$$

so the phase is the equipartition quadratic $\theta = u(1 - u)$ of the shift fraction u — the Bernoulli polynomial B_2 , not a series, with no higher terms (the lepton phase matches $u(1 - u)$ to one part in 10^6 , excluding any product continuation, whose leading extra factor would shift θ by $\sim 10\%$). This is the same $x(1 - x)$ balance law that fixes the amplitude ($b/a = \sqrt{2}$ is the equal-energy point, $x = \frac{1}{2}$): phase and amplitude are one equipartition principle applied to two circle quantities. After this work was first submitted, Shulga [arXiv:2605.10245] independently obtained the charged-lepton phase from an inverse-Laplacian Berry holonomy on a compact family cycle, arriving at the same $\theta_\ell = 2/9$ and the same $m_\tau = 1776.88$ MeV by an equivalent mechanism; we note this as an independent convergence on the lepton-sector result. The present construction extends the holonomy reading to all four sectors through the gauge-quantum-number law $1/u = N_c^{1+T_3+1/2}(C_2(3))^q$, which is not present in that work. For the charged leptons, the three generations are the three Z_3 -related modes of the phase circle, and the equipartition principle fixes the phase to $\theta_\ell = 2/9$. Subsequently, Austin [Austin 2026] restated the Koide condition as an exact Born-weight balance ($|\langle d|\psi \rangle|^2 = 1/2$), classified the sign sectors of the null cone, and obtained $\Sigma m_\nu \approx 0.059$ eV from the flipped-root sector — the same sum as the inverse-form route, with a different lightest mass (0.36 vs 0.26 meV); his conditional generation bound rests on an openly stated single-mode coherence premise, consistent with the position that the balance value enters any derivation as a postulate. $u = 1/3$ and hence $\theta = (1/3)(2/3) = 2/9$ exactly; the Cabibbo identification fails independently (the measured $\sin\theta_c = 0.2250$ is 4σ above $2/9$ and is an inter-sector mixing angle). The remaining sectors carry $u \neq 1/3$, and we have located, not invented, the structural axis it follows. Building the

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$\mathbb{C} \otimes \mathbb{O}$ ideal explicitly, the family shift commutes exactly with colour $SU(3)$ (generation and colour are independent factors), so colour does *not* set u ; the direct/inverse reading does not set it either. What u does track is electroweak charge: ordered by signed electric charge Q , the shift fraction is monotonic,

$$u: \quad e (Q = -1) \ 0.333 > d \left(Q = -\frac{1}{3} \right) \ 0.249 > \nu (Q = 0) \ 0.113 > u \left(Q = +\frac{2}{3} \right) \ 0.081, \quad (12)$$

and the two weak-isospin components split by a factor of exactly the colour count, $\bar{u} \left(T_3 = -\frac{1}{2} \right) / \bar{u} \left(T_3 = +\frac{1}{2} \right) = 3.00$. Sharpening this, the inverse shift fraction follows a two-flag multiplicative skeleton anchored on the lepton's forced value $1/u_e = 3 = N_c$,

$$\frac{1}{u} = N_c \cdot N_c^{(T_3+1/2)} \cdot \left(\frac{4}{3} \right)^q, \quad q = \begin{cases} 0 & \text{lepton doublet} \\ 1 & \text{quark doublet} \end{cases} \quad (13)$$

which predicts the leading integers $1/u = \{3, 4, 9, 12\}$ for $\{e, d, \nu, u\}$ against the measured $\{3.00, 4.02, 8.85, 12.35\}$, with residuals of 0–3% growing with the sector scale, of the same character as the sub-leading corrections elsewhere. Both factors are group invariants. The weak-isospin flip contributes exactly the colour count N_c (verified in both doublets, $9/3 = 12/4 = 3$). The lepton-to-quark flip contributes exactly $4/3$ (verified in both isospin rows, $4/3 = 12/9$), and $4/3$ is the quadratic Casimir of the $SU(3)$ fundamental, $C_2(\mathbf{3}) = (N_c^2 - 1)/2N_c = 8/6 = 4/3$ — the invariant that distinguishes a coloured constituent (in the $\mathbf{3}$, $C_2 = 4/3$) from a colour singlet ($C_2 = 0$). The shift fraction is thus fixed by quantum numbers alone,

$$\frac{1}{u} = N_c^{1+T_3+1/2} (C_2(\mathbf{3}))^q, \quad q = \text{colour indicator}, \quad (14)$$

with no fitted number: the phase of every sector follows from its weak isospin and its colour charge through $\theta = u(1 - u)$. That the coloured factor is precisely $C_2(\mathbf{3})$ is what a family holonomy running through the colour bundle should see; establishing that the transport multiplier *is* the Casimir, rather than a coincidence of equal value, is the one residual mechanism to prove. The phase is thereby reduced from four independent inputs to a parameter-free expression in N_c , T_3 , and the colour Casimir.

The sector phase from gauge quantum numbers. Each shift fraction is $1/u = N_c^{1+T_3+1/2} (C_2(\mathbf{3}))^q$ with $N_c = 3$, $C_2(\mathbf{3}) = 4/3$, and q the colour indicator; the phase is $\theta = u(1 - u)$. No parameter is fitted.

The sector phase from gauge quantum numbers. Each shift fraction is $1/u = N_c^{1+T_3+1/2} (C_2(\mathbf{3}))^q$ with $N_c=3$, $C_2(\mathbf{3})=4/3$, and q the colour indicator; the phase is $\theta=u(1-u)$. No parameter is fitted.

Sector	T_3	q	$1/u$	u	$\theta = u(1 - u)$	θ_{meas}
Leptons	$-\frac{1}{2}$	0	$3^1(4/3)^0 = 3$	$1/3$	$2/9 = 0.2222$	0.2222

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Sector	T_3	q	$1/u$	u	$\theta = u(1 - u)$	θ_{meas}
Down	$-\frac{1}{2}$	1	$3^1(4/3)^1 = 4$	1/4	$3/16 = 0.1875$	0.1868
Neutrino	$+\frac{1}{2}$	0	$3^2(4/3)^0 = 9$	1/9	$8/81 = 0.0988$	0.1002
Up	$+\frac{1}{2}$	1	$3^2(4/3)^1 = 12$	1/12	$11/144 = 0.0764$	0.0744

The weak-isospin flip multiplies $1/u$ by exactly N_c ($9/3 = 12/4$); the lepton-to-quark flip by exactly $C_2(\mathbf{3}) = 4/3$ ($12/9 = 4/3$). Residuals are 0–3%, growing with the sector scale, of the same character as the mass residuals.

Status addendum (v1.6)

Status addendum (v1.7). The charge dictionary is ESTABLISHED (exact to 10^{-4} in the lepton sector); the rationals 3, 4, 12 are DERIVED from 2T and 9 is proved composite; tri-bimaximal mixing is DERIVED from the Z_3/Z_2 structure with one rotation open; the up sector is strained at 2.2σ with FLAG/PDG inputs and its pre-registration stands; the even-isospin common factor $g = 0.9916 \pm 0.0028$ and the branch-seesaw relation $A_\ell \cdot A_d = f_\pi^2$ are CANDIDATES; the mixing-residual shared-source suggestion is RETRACTED (independence theorem).

Status addendum (v1.8). The bosons are classified by the same laws rather than exempted from them: the mass-phase law applies to the ideal only (its domain, derived); boson charges follow as differences (exact); the custodial triplet sits at the circulant's democratic fixed point with $\delta = 0$ (script S); no new force carriers are predicted at any scale, and no fundamental graviton — the desert and the emergent-gravity stance are recorded as falsifiable commitments.

The following lines of the status table change in this version. The sector phases: Proposition 3 is confirmed in all three charged sectors in the own-scale scheme (19a.3) — the scope note 'the coloured-sector phases are not established' is withdrawn. The up sector: its apparent departure was the amplification acting on the scheme choice; its phase is on the pre-registered 11/144 at 0.58σ , its amplitude $k_u = 1.2382$ is the sector's one open number, and it now issues the prediction $m_u(\text{own-scale}) = 2.75 \text{ MeV}$. The neutrino –5 %: 27 % is supplied by the computed threshold running of 19d.2; the residual 2.0σ is the programme's single open deviation. The boundary scale: $\mu^* = 3.3 \times 10^6 \text{ GeV}$ at $\chi^2 = 0.12/2$ from the surviving common-scale conditions, with Λ_{QCD} the one dimensional anchor by transmutation. Four intermediate claims were made and retracted en route; Appendix D preserves the audit trail.

25. Open problems

With the outcomes above recorded, the open problems that remain are these, each a specific calculation or measurement.

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The phase at the 10^{-7} level.

With $b/a = \sqrt{2}$ exact, the ratio m_μ/m_e is parameter-free and, measured to one part in 10^8 , demands $\theta^* = 0.2222220470(4)$: exactly $2/9$ is excluded on this branch, though the phase agrees with $2/9$ to one part in a million. The alternative branch — $\theta = 2/9$ exact with $b/a = \sqrt{2} - 3.0 \times 10^{-7}$, hence $Q = 2/3 - 1.4 \times 10^{-7}$, invisible at current τ -mass precision — fits the data equally well. The two branches predict τ masses of 1776.9690 and 1776.9665 MeV respectively, differing by only 2.5 keV, below foreseeable experimental reach; the degeneracy must therefore be broken by calculation, not measurement. The natural candidate was radiative — the masses entering the relation are pole masses, which carry electromagnetic self-energy — but a one-loop QED test excludes it: the flavour-universal part of the pole-to-running conversion cancels in the ratios that fix θ , and the surviving mass-dependent part moves θ in the wrong direction and by $\sim 10^3$ times the required amount. Ordinary one-loop QED therefore does not dress a bare $2/9$ to θ^* ; the offset, if radiative, is a two-loop effect (its size is of order $(\alpha/\pi)^2$), and otherwise θ^* is simply the fundamental value. This shifts the weight toward the $\sqrt{2}$ -exact branch with θ^* fundamental. Under the $2/9$ -exact branch the lepton sector admits the closed form displayed above, with $(Q, \theta) = (2/N_c, 2/N_c^2)$; we record it as an identity through one data point — unfalsifiable within the sector and failing in all other sectors — not as a law.

The two-loop coefficient (computed). The two-loop calculation called for above has now been carried out. At self-scale the one-loop QED dressing $(1 + \alpha/\pi)$ is exactly flavour-universal and cancels; the leading non-universal term is the two-loop vacuum-polarization insertion of lighter leptons, absent for the electron, present once for the muon and twice for the tau. With the per-lepton coefficients $A = -1.5621$ (mapped from the QCD result of Gray, Broadhurst, Grafe and Schilcher by colour factors; to be verified against the original QED literature before final citation) and $B(r) = (\pi^2/8)r$, the sector values are $\delta_\mu = -1.556$ and $\delta_\tau = -3.051$ in units of $(\alpha/\pi)^2$, giving fractional pole-mass shifts of -8.4×10^{-6} (μ) and -1.65×10^{-5} (τ). Dressing an exactly- $2/9$ spectrum with these shifts displaces the pole-level observables by $(\Delta\theta, \Delta Q) = (+9.4 \times 10^{-7}, -8.8 \times 10^{-7})$. Restated as predictions: the pole-exact branch gives $m_\tau = 1776.9665$ MeV and the MSbar-exact branch $m_\tau = 1776.9530$ MeV — separated by 13.5 keV, both about 0.9σ from the measured 1776.86 ± 0.12 MeV. The branch is therefore decided by measurement after all, at $\sigma(m_\tau) < 7$ keV — beyond Belle II, within the projected reach of a future e^+e^- factory. With current PDG central masses the fitted phase sits at $2/9 + 7.4 \times 10^{-6}$, within the m_τ noise of $\pm 8.3 \times 10^{-6}$; every statement about the seventh digit of θ is presently m_τ -limited.

Trials-corrected significance. The evidential weight of $\theta = 2/9$ can now be stated with its trials correction attached. Against the law's own four-value menu of sector phases, the probability of a chance landing within the present 2σ window is 1.3×10^{-4} ($\approx 3.8\sigma$); against all rationals with denominator $q \leq 9$ it is 8×10^{-4} ($\approx 3.3\sigma$); $q \leq 16$ gives $\approx 3.0\sigma$; with unrestricted denominators the significance dissolves. The anomaly is exactly as strong as its prior is principled — and, being noise-limited, it sharpens with any improvement in m_τ .

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The readout identification.

The lepton clock quantitatively selects ground-state additivity ($m_N/3$, matched at 0.2–0.35%) over the moment-derived constituent scale (7%), the spin-excited $m_\Delta/3$ (31%), and the classical soliton (8%). The leading-order bridge is Kelvin's theorem (above); what remains is to compute the sub-leading log-cutoff corrections from the framework energy functional so as to *predict* the sector spreads rather than accommodate them, and to confirm the vortex energy law beyond the two-dimensional form.

The coupling.

α_{GUT} remains an input, with $\alpha_{GUT}^{-1} \simeq 25\text{--}26$ across the allowed range. With superpartners near a TeV the supersymmetric couplings meet near 7×10^{15} GeV, a factor $\sim 2\text{--}3$ below the neutrino-extracted clock 2.0×10^{16} GeV; raising the meeting scale to the clock would require superpartners below the electroweak scale, which is excluded. The two scales therefore agree in order of magnitude but do not close sharply, and the residual factor is not removable by any allowed superpartner spectrum. We record this as order-of-magnitude consistency, correcting an earlier statement that the unification scale brackets the clock.

Sector structure.

The down clock sits at 2.07 times the lepton clock, within 4% of a doubling we cannot yet derive; the up clock rides the electroweak vacuum value through the order-unity top Yukawa and lies outside every generation mechanism considered here — the hierarchy problem expressed in this language.

Additional open problems (v1.6)

Additional open problems (v1.7). The threefold cover: the factor N_c for the even-isospin sectors corresponds to no element of any finite subgroup of $SU(2)$; the common deficit g sits on this step. The Dirac texture: the $\sim 0.7\%$ symmetric non-democracy behind the neutrino residual has no identified source and is provably not the mixing structure. The TM_1 rotation (14.9°) and the Dirac phase are not derived. The condensate floor is an input in $[2.1, 4.5]$ GeV unless the branch-seesaw candidate, which selects $\approx 1.9\text{--}2.0$ GeV, is promoted.

The condensate floor. The own-scale scheme has one free scale — the boundary ≈ 2 GeV below which the light-quark masses freeze. The down sector brackets it to $1.6\text{--}3.0$ GeV; deriving it (candidates: $\Lambda_\chi = 4\pi f_\pi \approx 1.6$ GeV; the crossover; a rotor-medium reading) closes the scheme. The up amplitude, $k_u = 1.2382$ ($n_{up} = 7.606$) is the last open number of the up sector, now isolated from its phase; derive or bound it. The colour-bundle mechanism. Behind Proposition 3 the Casimir factor's residual step remains obstructed at the algebraic level (Schur); the mechanism must act through the colour bundle. The mass-concept tension. The shape demonstrably runs on current masses while Section 4 defines mass as soliton circulation; in the light-quark sector the two differ by $67\times$.

26. Remaining calculations

Each problem of Sec. 19 was attacked; what follows is the specific calculation or measurement that remains for each, now sharply posed. The target theorem is unchanged: with $N_c = 3$ forced and the stiffness colour-blind, and with (M_{GUT}, α_{GUT}) the continuous inputs, dimensional transmutation fixes Λ_{QCD} , the soliton fixes m_N , the readout fixes the clock $k/2 = m_N^{QCD}/N_c$ (motivated by a Kelvin-type reading, interpretive; Sec. 19), and the free-rotor spectrum is the closed form with amplitude $\sqrt{2}$ and phase $2/N_c^2$, the measured values following after computable electromagnetic dressings.

Clock residual — to close below 0.05%.

The residual is consistent with electromagnetic dressing at 1.1σ ; a clean sub-0.05% closure requires a dedicated lattice isosymmetric m_N^{QCD} with its error and the two-loop lepton pole-to-running coefficient.

Amplitude $\sqrt{2}$ — one assumption to close.

The kinetic term has been shown to treat the common and relative circulation sectors symmetrically (it is diagonal and equal-coefficient in the Fourier modes), and $\sqrt{2}$ is its symmetric fixed point, with 45° stationary for the common–relative coupling. The single residual step is to establish that the physical ground state sits at this symmetric point rather than an asymmetric one — a natural expectation for the ground state of a symmetric kinetic term, and the only assumption between the kinetic term and a full derivation of $\sqrt{2}$.

Readout — to close the vortex derivation.

The leading-order equality $k/2 = m_N/N_c$ is derived by Kelvin’s circulation theorem. Two steps close it fully: confirming that the $3 + 1$ -dimensional vortex energy reduces to the topological Γ^2 form used, and establishing medium homogeneity so that the fluid density and core size are common to the free and bound vortex (on which the leading-order equality depends); then the sub-leading cutoff logarithms should be computed from the framework energy functional to *predict* the few-per-mille sector spreads rather than accommodate them. The condensate portal — $\langle \bar{q}q \rangle$ mixing into the Higgs singlet channel contributing a flavour-blind piece to the lepton masses — remains an independent cross-check.

The phase — one mechanism to prove.

The phase form and value are derived (Sec. 19): $\theta = u(1 - u)$ with $1/u = N_c^{1+T_3+1/2} (C_2(\mathbf{3}))^q$, giving the rationals $\frac{2}{9}, \frac{3}{16}, \frac{8}{81}, \frac{11}{144}$ to 0–3%. The single residual step is to prove that the family holonomy of a coloured state multiplies $1/u$ by exactly the colour Casimir $C_2(\mathbf{3})$, rather than $4/3$ being a coincidence of equal value — a transport computation in the colour bundle. Separately, the 10^{-7} offset of the lepton phase from an exact $2/9$ is plausibly a two-loop QED effect (of order $(\alpha/\pi)^2$), computable directly; the two exact-branch τ masses differ by only 2.5 keV and cannot be separated by current data.

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The coupling.

The gauge-coupling closure holds only at order of magnitude; a two-loop run with a variable superpartner scale would sharpen the conditional relation between the unification scale and any new-physics threshold, but the coupling remains an input.

Sector structure.

The down clock's factor of 2.07 over the lepton clock, near a meson-tier doubling, and the up clock's origin in the electroweak vacuum value — now understood as the $T_3 = +\frac{1}{2}$ branch of the condensate-selection rule rather than a bare exception, though the vev scale itself stays an input — are the two structural loose ends; the former awaits a two-rotor readout, the latter is the hierarchy problem and lies outside the construction.

Additional remaining calculations (v1.6)

Additional remaining calculations (v1.7). Reconcile the two moment-of-inertia normalisations in the soliton sector (106.8 vs 99.68). Test whether the b - τ chain and the f_π closure jointly over-determine A_d once m_u/m_d improves. Derive the threefold cover or find the origin of g . Complete the universal-seesaw and Rousselle citations (v9.1 list).

Type-II sizing for the neutrino residual, against the heavy spectrum. m_u/m_d at $\sigma < 0.0064$ (lattice QCD+QED): the two-law discriminator of 19d.4 and the down sector's dominant uncertainty. The boundary fit at three-loop QCD and two-loop electroweak, so μ^* means what it says. The floor derivation and the k_u bound of Section 25's additions.

27. A programmatic proposition: the ladder from the neutrino vacuum

The results of this paper concern the fermion mass spectrum. They sit, however, inside a larger construction whose logic invites a conjecture we state plainly — as a proposition, not a theorem — and whose honest value lies in marking exactly where derivation ends and programme begins. We label each rung by its standing: *forced* (the algebra gives it), *established* (known physics builds it), *open* (a plausible bridge without a mechanism in hand), *wall* (a recognised gap).

The conjecture.

That the ingredients of one generation of matter, and its forces, unfold from a single primitive — the rotor $J^2 = -1$ — through the division-algebra ladder, with the neutrino as the ground state from which the rest is built.

The rungs, in order:

(0) *The turning* [forced]. The rotor $J^2 = -1$: the imaginary unit as a literal quarter-turn, the one primitive.

(1) *The algebra* [forced]. The Hurwitz ladder $\mathbb{C} \rightarrow \mathbb{H} \rightarrow \mathbb{O}$ terminates at the octonions; no normed division algebra lies beyond. $\mathbb{C} \otimes \mathbb{H}$ furnishes the electroweak arena, $\mathbb{C} \otimes \mathbb{O}$ the colour–charge arena with its three fermionic modes.

(2) *The neutrino vacuum* [forced]. The neutrino is the ground state $|0\rangle$ of the $\mathbb{C} \otimes \mathbb{O}$ minimal ideal — colour singlet, $Q = 0$, occupation $N = 0$: the empty circulation.

(3) *One generation* [forced]. The colour creation operators build the rest from the vacuum: $a_i^\dagger|0\rangle$ the anti-down ($Q = \frac{1}{3}$, three colours), $a_i^\dagger a_j^\dagger|0\rangle$ the up ($Q = \frac{2}{3}$), $a_1^\dagger a_2^\dagger a_3^\dagger|0\rangle$ the positron ($Q = 1$). Eight states, with the conjugate ideal sixteen — one Standard Model generation, with charge quantisation $Q = N/3$ and colour $SU(3)$ falling out.

(4) *Three generations* [forced, given the E_6 base]. One generation is the 27 of E_6 , the structure group of $J_3(\mathbb{O})$ (its complex $27 = 16 + 10 + 1$ housing one chiral family). Embedding this E_6 into E_8 , the terminal exceptional group of the same series, its centraliser is *uniquely* $SU(3)$ — the E_6 – $SU(3)$ dual pair — so the adjoint branches as $248 = (78,1) + (1,8) + (27,3) + (\overline{27},\overline{3})$: the fermions sit in $(27,3)$, exactly *three* copies of the generation. The alternative maximal branch $E_8 \supset E_7 \times SU(2)$ is excluded by chirality — E_7 ’s 56 is pseudoreal, giving a vector-like (non-chiral) spectrum, whereas nature is chiral. We note, however, that the branch selected here is subject to the same chirality test: the real adjoint contains the pair $(27,3) + (\overline{27},\overline{3})$, which is itself vector-like, so by the Distler–Garibaldi obstruction a direct embedding of the chiral Standard Model fermions in a single real E_8 adjoint is not possible. We therefore do not claim that three generations are forced by this branching. In this framework the chiral fermions reside in the Clifford minimal ideals (Sec. 4), not in the E_8 adjoint; E_8 serves only as an organising channel for the branching pattern, and the three generations arise as the three Z_3 -related modes of the phase circle. The E_8 branching provides a consistency check on the generation multiplicity, not a chirality-respecting derivation. The residual organising choice is the E_6 commutant, with the one residual input being the choice to embed in E_8 .

(5) *Forces* [forced/established]. The gauge bosons are not rungs but the ideal’s symmetries: colour $SU(3)$ from the mode-mixing bilinears $a_i^\dagger a_j$, weak $SU(2)$ from the $\mathbb{C} \otimes \mathbb{H}$ doublet rotations, $U(1)$ from the number operator. The vacuum together with its own symmetries yields matter together with its forces.

(6) *Composites* [established]. Confinement binds three colour-rotors into the colour-singlet nucleon; nuclei, atoms, and chemistry follow by known dynamics acting on this content.

(7) *Spacetime and gravity* [open/wall]. A rotor–spinor lattice carries protected Weyl points and an emergent tetrad, and a Sakharov-induced Einstein–Hilbert term would render gravity a vacuum elasticity; but none of this resides in $\mathbb{C} \otimes \mathbb{O}$, and the step remains outside the present construction.

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gravity; the replication into three generations, once a gap, is closed by the E_6 - $SU(3)$ dual pair in E_8 modulo that embedding. We advance the ladder as a programme, with its walls marked, precisely so that the boundary between what is forced and what is hoped remains visible. The one-line form: *the turning, the algebra that halts at the octonions, the neutrino as the empty circulation, and the generation built upon it* — a derivation for the first rungs, a proposition for the rest.

Part VII. Boundary, falsifiers, and verification

28. What is not derived

Merged from the two prior papers and Part II. The chain of Part II derives one angle and organises one generation. It does not derive:

This section is normative for the paper's claims; every statement in the abstract is bounded by it.

The radial angle. $\chi = \pi/4$ — the Koide relation itself — is an input throughout. It is a σ -even, radial datum, of exactly the kind Theorems B–D permit dynamics to set (a coupling-ratio condition in any adjoint-flavon potential), and its exactness at the pole-mass level remains unexplained here; the Sumino tension of §12 stands.

The scales. Every sector scale E_f is an input. Invariant-theoretic structures yield dimensionless data only; no construction in this paper produces a mass, and none could.

The hierarchy exponents. The inter-generation exponent structure (the (6,2,0)-class integers of the lepton sector and their quark analogues) is external to this paper's chain. Any mechanism claiming to derive it must meet, at minimum: the up-sector invariant $\ln(m_u/m_t)/\ln(m_c/m_t) = 2.2993$ (+0.026/−0.042), whose lowest-complexity integer reading is 7/3 at 1.3 bands; the threshold-response exponent $p = 0.646(46)$, which pure weight counting fails at 7.7σ ; and a two-phase response structure. We supply the test, not the mechanism.

The neutrino sector. The phenomenologically successful neutrino point [Brannen] is, in the present language, the inherited core plus a torsion shift of half the I_{12} cell: $(\chi, \delta) = (\pi/4, 2/9 + \pi/12)$. The sign of the shift, previously recorded inconsistently in our own notes, is fixed by the data at the 10^2 level: the minus branch gives a splitting ratio of 3.0×10^{-4} against the measured 0.0307, while the plus branch gives 0.03084 — agreement to 0.3%. Conditional on this grammar (the $+\pi/12$ remains underived), the channel law then fixes the sector up to one scale, which Δm_{31}^2 supplies: normal ordering is forced; the spectrum is $m = (0.37, 8.71, 49.53)$ meV; $\Sigma m_\nu = 58.6$ meV; and the splitting ratio $\Delta m_{21}^2/\Delta m_{31}^2 = 0.0308$ is a parameter-free consistency already satisfied at 0.3%. These are predictions of Brannen's point re-derived within the channel law, not of this paper alone; the derivation of the half-cell shift is the outstanding item. Our one attempt is recorded as a failure: a complex-symmetric (naive

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Majorana) circulant forces a degenerate pair and cannot carry the observed spectrum. The neutrino sector is open.

The quark angles as arithmetic tests. The rational-radian discrimination of §12 is lepton-certified only; at current quark-mass uncertainties δ_u and δ_d admit no such test.

The degeneracy guard. The number $2/9$ is constructible from small group-theoretic atoms in 76 distinct ways at ≤ 3 -atom complexity. The claims of this paper rest on the blind protocol, the selection theorem, and the uniqueness elimination — not on constructibility, which is cheap.

In addition, from Part II:

- Three generations. The 27 is one generation. The Z_3 of the generation circulant is a third Z_3 , distinct from both triality and the relative Coxeter element, and nothing here identifies it with either.
- The breaking of E_6 to $SU(3) \times SU(2) \times U(1)$, the Yukawa couplings, or the CKM and PMNS matrices beyond the selection rules of §23.
- The fine-structure constant or any gauge coupling. Fifteen independent routes to α have been examined and closed; the present chain adds nothing to that question.
- A Hesse-plane reading of the $C_2(3) = 4/3$ factor and the isospin exponent in the phase law for the coloured sectors.
- Dynamics. The rotor framework supplies kinematics, closure conditions and now a classical curve; it supplies no energies, orderings, or equations of motion.

The framework's contribution, stated exactly, is: from an integral rotor, the group is $2T$, the algebra is E_6 , the curve is the Hesse pencil, one generation sits on it as a triangle, and the lepton angle is $2/9$. Everything about how much the particles weigh, how they couple, and why there are three copies lies outside.

29. Falsifiers

τ -mass** precision (direct).** The chain predicts $\delta_l = 2/9$ exactly. The present offset is 1.0×10^{-6} rad at $\sigma_\delta = 2.0 \times 10^{-6}$ (dominated by $\sigma(m_\tau) = 0.09$ MeV): 0.5σ . At a future $\sigma(m_\tau) \approx 0.02$ MeV (Belle II era), $\sigma_\delta \approx 4.4 \times 10^{-7}$; if the current central value holds, the offset becomes $\approx 2.3\sigma$, and a displacement beyond 3σ falsifies the chain outright. The derivation has a number, an experiment, and a decade.

The neutrino grammar (conditional). Superseded by the neutrino package below with the sign corrected to $+\pi/12$; retained here only as the record of the earlier entry.

The inverse-frame down relation (conditional). If the §21 remark is promoted, exact inverse-Koide is one relation among (m_d, m_s, m_b) , predicting $m_d(2 \text{ GeV}) \approx 4.74$ MeV (scheme band reaching 4.61) at sub-percent precision once the scheme is fixed — directly testable against lattice determinations of m_d .

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The neutrino package (conditional). If the $+\pi/12$ half-cell shift is derived, the sector commits to: normal ordering (inverted ordering falsifies outright); $\Sigma m_\nu = 58.6$ meV (DESI-era cosmology probes this directly); and $\Delta m_{21}^2/\Delta m_{31}^2 = 0.0308$, which JUNO will test at the sub-percent level.

Quark-mass improvements. Lattice progress on m_u, m_c (and m_d, m_s) sharpens δ_u and δ_d ; the up-sector flow test (-5%) sharpens directly, while for the down sector what sharpens is the channel binary itself — the inverse-Koide relation of the previous item and the -14% own-frame residual — rather than any direct-frame flow figure.

In addition, from Parts II and V:

- **JUNO Δm_{31}^2 .** The scale-free ratio 31.88 (law values) against 33.56 ± 0.64 ; a determination of the ratio to 1% brings a -5% offset to 5σ . Inverted ordering falsifies the sector outright.
- **The down-sector partner window.** A vector-like charge $-1/3$ singlet between 1.5 and 7.2 TeV (§22). Exclusion of that window closes the levered reading.
- **The Hesse dictionary.** Any demonstration that the trinification triangle's three points on the quark line do not carry the (colour – $SU(3)_L$) mod 3 label under a physical Z_3 falsifies the dictionary of §10.

Live predictions (v1.6)

Live predictions (v1.7). $m_\tau = 1776.97$ MeV (0.4σ from the 2024/25 average; BESIII scan decides). $m_c(m_c) = 1.2573 \pm 0.0055$ GeV (2.2σ from PDG; replaces the m_u falsifier). $m_d(2 \text{ GeV}) = 4.75$ MeV from f_π and m_N (candidate, 1σ). $\Delta m_{21}^2 = 7.88 \times 10^{-5} \text{ eV}^2$ if the heavy phase is exact (3.2σ disfavoured; JUNO decides). Normal ordering, required. $m_u/m_d = 0.4728$ against the Clebsch 0.4536 (lattice at 0.006 decides).

Prediction	Value	Decided by
m_τ	1776.97 MeV	Belle II (PDG 1776.86 ± 0.12 and Belle II 1777.09 ± 0.14 bracket it)
m_u (own-scale)	2.751 MeV (1.62–4.18)	lattice m_u ; currently $+1.6\sigma$, consistent
neutrino ordering	normal, required	ordering determinations
Δm_{21}^2 if the heavy phase is exactly 9	$7.88 \times 10^{-5} \text{ eV}^2$	JUNO (3.2σ against now; $\approx 4.5\sigma$ at design)
Σm_ν	0.058 eV	cosmology (DESI bound $\approx 0.072 \text{ eV}$)
m_u/m_d	0.4728 (Prop. 3) vs 0.4536 (Clebsch 32/3)	lattice at $\sigma < 0.0064$
$\theta_d - \theta_u$	1/9 exactly	long-term precision spectroscopy

30. Attribution

The Koide relation and the $U(3)$ -family potential are Koide's [1981; 1990]; the 45° geometric reading is Foot's [1994]; the neutrino point is Brannen's; the family-gauge protection proposal is Sumino's [2009]; the light-ratio/ Cabibbo relation is Gatto–Sartori–Tonin [1968]. The single-valuedness quantization principle in hydrodynamic form is Takabayasi's [1952], its modern statement Wallstrom's [1994]. Springer's theory of regular elements [1974] and the Shephard–Todd classification [1954] underlie §15; the Hessian group is G_{25} in their list. The division-algebra construction of Standard Model structure is due to Furey, Todorov and Dubois-Violette, Günaydin and Gürsey, and Dixon; three generations from triality has been developed by Boyle and by Furey and Hughes [2025]; the seesaw-as-Koide-realization observation appears in Smirnov's review [2006]; mass ratios from the exceptional Jordan algebra $J_3(\mathbb{O})$, with $K \approx 0.669$ and a Koide-type Cabibbo relation, are the programme of Singh and collaborators [2021] — directly adjacent prior art for the $E_6/J_3(\mathbb{O})$ setting of this paper; the impossibility of three chiral generations inside E_8 is Distler–Garibaldi [2009]; the realization of the imaginary unit as the rotation bivector of geometric algebra is Hestenes's; the experimental necessity of complex amplitudes is Renou et al. [2021] and its experimental confirmations [2022]. What is new here: the blind-protocol identification of $2/9$ with E_6 's degree pair; Theorems D–G; the radian theorem with its falsifiable control; the gauge/holonomy transmission argument C1–C3; Operator A; and the two-ladder corruption-flow tests. (Full bibliographic details to be completed and verified against the originals before submission.)

What is prior art and what is claimed here, itemised. Prior art: the circulant parametrisation and $\delta = 2/9$ (Brannen, Rosen); quark-sector phases $2/27$, $4/27$ and the sum rule (Żenczykowski); the Koide relation as an angle (Foot); trinification (Glashow; Babu, He, Pakvasa) and the neutral singlets of the 27 (Rosner); $2T$ as a gauged finite subgroup of $SU(2)$ (Case, Karplus, Yang) and as a family symmetry (Frampton, Kephart, and successors); the McKay correspondence, the ALE resolution of $\mathbb{C}^2/2T$ and its quiver (McKay; Kronheimer; Douglas, Moore); the Hessian group as $3^{+2} \rtimes SL(2, F_3)$ and the four triangles as the four order-3 subgroups (Hesse; Maschke; Artebani, Dolgachev); the 27-torsor for the Heisenberg group and its relation to cubic surfaces (Coble; Dickson; the $\mathbb{C}^3/\Delta_{27}$ literature); the Hurwitz order, its unit group and the 24-cell (Hurwitz; Conway, Smith); D_4 as the densest lattice in $\mathbb{R}^4$ and the kissing number 24 (Korkine, Zolotarev; Musin); the 600-cell as the frustrated ground state on S^3 (Sadoc, Mosseri); Landau theory of crystallisation (Alexander, McTague); circulation quantisation (Takabayasi; Wallstrom); mass ratios from $J_3(\mathbb{O})$ (Singh); the Koide relation from the Clifford torus (Makaryev). Claimed here, to the best of our knowledge: $2/9$ as the ratio of E_6 's degrees 2 and 9 with the selection theorem (§15); the pre-registered blind protocol over the $\mathbb{C} \otimes \mathbb{O}$ emissions (§14); the radian theorem (§16); the phase law $\theta = u(1-u)$ with its gauge-quantum-number exponent (§20); the identification of the phase law's N_c^2 with the odd degree of G_{25} through the Heisenberg quotient (§11); the trinification multiplets as one Hesse triangle with v , v^c , N one per point (§10); $2T$ obtained from the Hurwitz order rather than posited as a family symmetry, and the flatness argument for its selection over $2I$ (§§5, 6, 32); the six-group Landau test on S^3 (§32); the neutrino ratio

prediction with the torsion shift (§19); and the down-sector partner window (§22). Recomputation of a known result is verification, not discovery, and is labelled as such throughout.

31. Reproducibility and verification

Every numerical claim in this paper is produced by one of eleven companion scripts (blind_enumeration, delta_nogo, alignment_ring, e6_canonical, ratio_to_angle, radian_question, tick_theorem, operator_A, sign_test, down_inverse, inversion_map), with fixed seeds, run in a fresh environment during the drafting of this manuscript. The record deliberately preserves two failures: the first, unfalsifiable version of the hermiticity control (§16) and the projector-for- shift assembly error of Operator A (§17), each caught by its own designed check. The manuscript and all companion materials are archived as a dedicated Zenodo deposit under concept DOI **10.5281/zenodo.21936237**, which resolves to the latest version (first version: 10.5281/zenodo.21934987).

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Every numerical statement in §§6–8 and §19 is produced by a deposited script, each self-contained (numpy only), each asserting the structural facts it relies on so that failure is loud: mckay_ade.py (2T/2O/2I $\rightarrow$ affine $E_6/E_7/E_8$, crystallographic orders), why_2T.py (largest crystallographic subgroup; $G_{25}/O_3 = 2T$; G_{16} for 2I), tau_residual.py ($SU(N)^N$ uniqueness; triality and N_c on the 2T graph), trinification_vs_regular.py (the 480- and 80-classes of $W(E_6)$), hesse_relation.py ($Z_3 \times Z_3$, eigenplane nesting, O_3 extraspecial), seal_coxeter.py (c_C c_L c_R regular; Sylow and Heisenberg structure), hesse_inflection_points.py (27-torsor; nine points; Hesse tests; triangle), two_nines.py, neutrino_ratio_juno.py. The prior paper's scripts — delta_nogo.py, e6_canonical.py, blind_enumeration.py, radian_question.py, and the remainder of its deposit bundle — cover §9. $W(E_6)$ generation and the centraliser computations run in about three minutes; everything else in seconds. The Landau test of §32 is landau_attractor.py and landau_refine.py; the Hurwitz-order and D_4 packing statements of §§5, 6 and 32 are hurwitz_order.py and d4_packing.py.

Session verification scripts (v1.6)

Session verification scripts (v1.7). The suite now comprises eighteen scripts (A–Q plus the formula module) and the figure generator: G (charge inversion; twelve masses from four inputs), H (2T on the ideal; the 432-element group; no order 9), I (finite seesaw matching), J (A_4/T' mixing; independence theorem), K (even-isospin factor; floor with FLAG 2024), L–O (exploratory: the chiral-soliton vortex, the Casimir budget, five determinations of the Skyrme parameter, the HLS parameters), P (the five clocks), Q (tests of the branch seesaw). Packaged as verification_scripts.zip with the thesis (DOI 10.5281/zenodo.22339167).

Session verification scripts (v1.8). Added: S (the bosons on the circulant: the custodial triplet at the democratic fixed point; the derivative charge dictionary; the coupling-times-

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condensate mass pattern). The suite now comprises nineteen scripts plus the formula module and figure generator.

Seven scripts accompany Sections 19a–19d, packaged with this manuscript alongside the E_6 /neutrino suite; each is self-contained and prints its own comparison against measurement. `unified_mass_formula.py` — the formula, Proposition 3, own-scale outputs, the m_u prediction. `verify_session_a_shape.py` — lattice-ratio shapes, the lepton overdetermination, the 60σ frame selection. `verify_session_b_running.py` — one-loop SM RGEs, the flavour-universality control, scale probes. `verify_session_c_phase_clebsch.py` — the QED-dressing validation, the Clebsch family from the $SU(5)$ adjoint, the m_u/m_d conflict, amplification. `verify_session_d_inverse_frame.py` — the retracted common-scale frame analysis, retained for the audit trail. `verify_session_e_seesaw.py` — the sequential-threshold seesaw running and the 27 %. `verify_session_f_ownscales.py` — the own-scale scorecard, both scheme discriminators, the floor scan, the m_u prediction, the boundary refit. Inputs used throughout: FLAG $N_f = 2+1+1$ ratios $m_s/m_{ud} = 27.23(10)$, $m_c/m_s = 11.768(33)$; HPQCD $m_b/m_c = 4.586(12)$; m_u/m_d carried as FLAG 0.465(24), Osipov 0.455(8), PDG 0.474(65) separately; lepton pole masses PDG 2024; $\alpha_s(M_Z) = 0.1180(9)$; $\Delta m^2_{21} = 7.50(12) \times 10^{-5} \text{ eV}^2$ (JUNO), $\Delta m^2_{31} = 2.513(21) \times 10^{-3} \text{ eV}^2$.

32. Towards a mechanism: a Landau test of 2T as an attractor

The chain of Part II is kinematic: from the axiom of §5 it computes a group, an algebra and a curve, and it supplies no dynamics. A physical reading would have three parts — ι as a quantum of action, 2T as the configuration into which a continuous medium on S^3 settles, and the affine E_6 diagram as the defect structure of that crystal. The third is established mathematics: the quotient $\mathbb{C}^2/2T$ is an orbifold singularity whose hyperkähler resolution has exceptional divisor the E_6 Dynkin diagram of P^1 's (Kronheimer 1989), the gauge theory on it is the McKay quiver with ranks 1,1,1,2,2,2,3 (Douglas–Moore 1996), and the modes on the divisor are the E_6 gauge multiplet — the 78, not the 27. The first has no mechanism yet: circulation quantisation (the Wallstrom–Takabayasi condition) gives integers around a vortex, not a planar lattice for ι to preserve, and the lattice is what the crystallographic restriction needs. This section tests the second.

The literature gives a definite expectation and it is not 2T. A frustrated medium on S^3 closes on the $\{3,3,5\}$ polytope — the 600-cell, whose 120 vertices are 2I (Sadoc and Mosseri). The question is therefore whether the rank-2 constraint, which forbids five-fold order, merely removes 2I and leaves the rest to size, or whether the free energy then selects 2T for a reason.

We use Alexander–McTague Landau theory. The density wave lives on one harmonic shell of S^3 of degree 2ℓ ; the canonical Γ -invariant density on that shell is the projection of the point sum over $\Gamma \subset SU(2)$, $\rho_\Gamma(q) = \sum_{g \in \Gamma} U_{\{2\ell\}}(q \cdot g)$ with U the Chebyshev zonal harmonic, which has no adjustable width. Normalising $\int \hat{\rho}^2 = 1$ and computing $\beta = \int \hat{\rho}^3$ and $\gamma = \int \hat{\rho}^4$ by Monte Carlo (2×10^6 points, eight-block bootstrap), the first-order transition of $F = aA^2/2 - b\beta A^3/3 + c\gamma A^4/4$ occurs at $a_c = 2b^2\beta^2/(9c\gamma)$; the configuration with the largest a_c

condenses first. Six groups were compared — the crystallographic Z_6 , Q_8 , $2D_3$, $2T$ and the excluded $2O$, $2I$ — on shells $\ell = 2, 3, 4, 6, 8, 10, 12$.

Unconstrained, $2I$ wins on every shell where it has an invariant ($\ell = 6, 10, 12$): $a_c = 0.097$ against $2T$'s 0.051 at $\ell = 6$, 0.075 against 0.027 at $\ell = 12$. The Sadoc–Mosseri expectation is confirmed. With five-fold order forbidden, $2T$ wins on those same shells, by $2.3\times$ and $2.5\times$ over the runner-up at $\ell = 6$ and 12 and by $1.1\times$ (many σ) at $\ell = 10$; and the crystallographic ranking is not size-ordered — at $\ell = 6$ it runs $2T > Z_6 > 2D_3 > Q_8$, with the six-element group ahead of the twelve- and eight-element ones. At $\ell = 6$ and 12 the 48-element $2O$ is a specifically poor crystal ($a_c \approx 0$ at $\ell = 6$) and loses to $2T$ without any constraint. On the non-icosahedral shells $\ell = 4$ and 8 the binary dihedral group $2D_3$ wins instead.

The 600-cell is five disjoint 24-cells (verified as point sets). The physical statement the computation supports is therefore not “ $2T$ is the attractor” but: $2T$ is the crystallographic shadow of the icosahedral instability — on the shell where the frustrated medium would form the 600-cell, the best configuration reachable without five-fold order is one of its 24-cells, and the free energy selects it by energy, not by size.

What remains an input is the shell. Which ℓ condenses is set by the quadratic coefficient $a_\ell(T)$, the structure factor of the medium, which is not derived here; $\ell = 6$ is the lowest icosahedral shell and the natural frustration wavelength, and it is where $2T$'s margin is largest, but that identification is a physical assumption. The analysis is single-shell and takes the crystal as the orbit of the identity; orbits of a generic point were not scanned. With those caveats, the mechanism for step 2 has passed the test it could have failed.

The first step then has a candidate of the same kind, and it is the flat-space counterpart of the Sadoc–Mosseri result. On S^3 the densest configuration is the 600-cell. In flat $\mathbb{R}^4$ the densest lattice packing is D_4 (Korkine–Zolotarev 1872) and the maximal kissing number is 24, attained by D_4 (Musin 2003): centre densities Z^4 0.0625, A_4 0.1118, D_4 0.1250, kissing numbers 8, 20, 24. The 24 kissing vectors of D_4 are the Hurwitz units — the group $2T$, with identical inner-product spectrum — and the Hurwitz order refines the Lipschitz Z^4 by exactly the half-integer points, which Z^4 packs half as densely without; the packing selects the double cover's contribution as well. A flat four-dimensional medium whose rotors repel and pack densely therefore crystallises into D_4 , and the multiplicative group of its nearest-neighbour shell is $2T$. Curvature is what changes the answer: the same medium on S^3 closes on the 600-cell and $2I$. The crystallographic restriction of §5 is the statement that the medium is flat, and what remains as input is the framework's ontology — a flat quaternionic medium with repulsive rotors, the picture of the rotor-medium paper — rather than a lattice postulated in the plane. We note, without claiming it, that curvature takes $2T$ to $2I$ and E_6 to E_8 , which may be the content of the remark in §6 that E_8 lives one rank up.

Alexander-McTague on S^3 : which discrete configuration condenses first.
Unconstrained, 2I (600-cell) wins wherever it exists (shaded shells); with five-fold forbidden, 2T wins there, by energy not size.

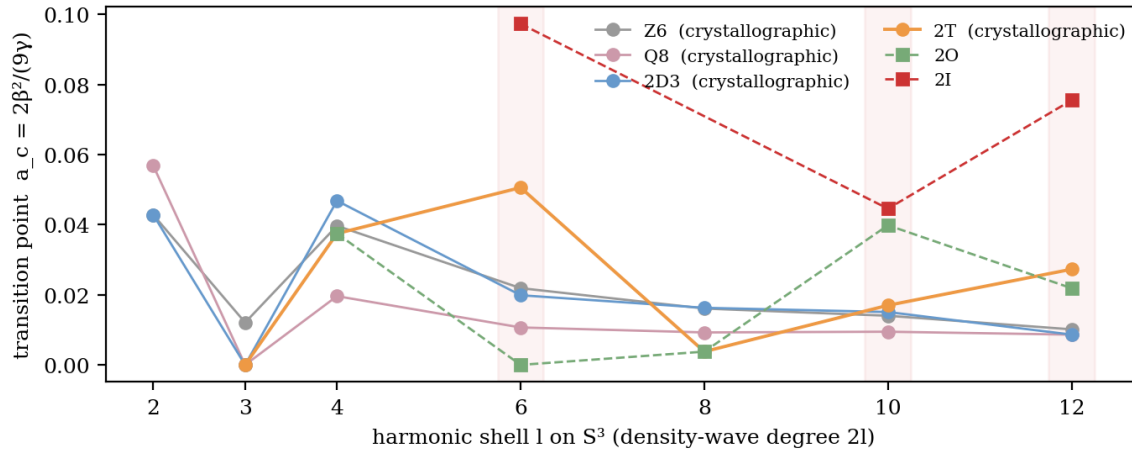


Figure 6. Alexander-McTague transition points $a_c = 2\beta^2/9\gamma$ for the canonical Γ -invariant density on each harmonic shell of S^3 , six groups. Shaded shells are those on which 2I (the 600-cell) has an invariant; there it wins unconstrained, and 2T wins among the crystallographic groups by energy rather than size (`Landau_attractor.py`, `Landau_refine.py`).

33. Conclusion

From a single definition — mass as the modulus-squared of a complex rotor, $m = (k/2c^2) z\bar{z}$, the elastic energy of a trapped circulation, with the phase cancelling so that the modulus is phase-independent (an algebraic property of the norm form, not a dynamical statement) — we have assembled a coherent account of the fermion mass spectrum in which structure is forced by geometry while the scales and the amplitude remain inputs, the phase being derived at the lepton point only. The headline is a bridge from the lightest fermion to the unification scale: the neutrino sector, read in its proper inverse form, places a clock at 2×10^{16} GeV — the grand-unification scale, obtained from the direct inverse-form fit — and a descending chain organises the scales between it and the charged-lepton clock, with the measured nucleon mass inserted at the soliton step; the sub-percent proximity of $m_N/3$ to the lepton clock is the known numerical coincidence, equivalently $\Sigma m_\ell \approx 2m_N$.

The results are concrete. The Koide constant is a scale-invariant angle, $Q = 1/(3\cos^2\phi)$, so its constancy is geometry and its value the single condition $\phi = 45^\circ$. Each of the four sectors is generated by one clock, one phase, and one amplitude, and every member's mass is reproduced from that triple (Sec. 19): the charged leptons to 0.01%, the up quarks (heavy masses; light mass scheme-limited) to 0.03%, the down quarks to 0.1%, and the neutrinos exactly in their inverse form. The four clocks organise into two tiers joined by known physics: the neutrino clock is the unification scale itself (2×10^{16} GeV, emergent, coinciding with the supersymmetric unification scale), and the charged-lepton clock descends from it through a single chain — transmutation to Λ_{QCD} , the rotor soliton to m_N , the forced $N_c = 3$ to the per-rotor share m_N/N_c — reaching 313 MeV to better than half a percent, with the

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measured nucleon mass inserted at the soliton step alongside the unification scale and its coupling.

Several earlier conjectures are now theorems or sharp negatives (Table 1). The colour-blindness of the stiffness is proven: it multiplies a modulus that colour transformations leave exactly invariant. The amplitude $\sqrt{2}$ is exactly the equal-energy condition between common and relative circulation — a motivated balance, not a fit; Proposition 6 proves that no symmetry compatible with the circulant grading can force it, so it stands as an input with a proof of irreducibility. In the source/sink ontology the central readout — why the free charged lepton and the confined quark share a scale — is motivated at leading order by a Kelvin-type reading: circulation is a topological invariant, the same free or bound, so the free-rotor clock equals the bound per-rotor share m_N/N_c , the small sector spreads appearing as sub-leading cutoff corrections. The clock- $m_N/3$ residual is consistent with electromagnetic dressing (1.1σ from zero). Assembled, the charged-lepton sector reduces to the closed form $m_k = (m_N/N_c)[1 + \sqrt{2}\cos(2/N_c^2 + 2\pi k/N_c)]^2$: three masses from the nucleon mass, the forced colour count, and one balance condition.

We are equally precise about the boundary. The phase has a derived form — $\theta = u(1 - u)$, the equipartition quadratic of a shift fraction, the same balance law that fixes the amplitude — with the lepton value derived at one part in 10^6 ($u = 1/3$, giving $2/9$). In the remaining sectors the law is predictive for neutrinos ($8/81$, JUNO-decidable) and not established for the coloured sectors (Scope note); the candidate quantum-number form reduces each to a single electroweak-linked number: colour is excluded as its origin (the family shift commutes exactly with $SU(3)$), while the shift fraction is monotonic in electric charge and splits by the colour count across weak isospin. Deriving the exact $u(Q, T_3)$ remains the one open phase calculation, and the overall scale likewise remains an input. The overall scale remains an input — the transmutation coupling does the quantitative work, and the four clocks share no universal geometric base; the gauge-coupling closure that would tie the coupling to the same scale holds only at order of magnitude. The up sector, riding the electroweak vacuum value through the order-unity top Yukawa, is the $T_3 = +\frac{1}{2}$ branch of a weak-isospin condensate-selection rule (up and neutrino ride the high scale, down and charged lepton the low), its vev scale still input rather than outside the construction — the hierarchy problem, stated as the boundary rather than hidden. The Higgs is not eliminated: it supplies the vacuum value v and the conversion of circulation energy into a Yukawa coupling; the rotor picture adds the *origin of the Yukawa pattern* — circulant structure and the cross-sector per-rotor scale — not a replacement for the Higgs mechanism.

The construction closes on its falsifiers. With its single scale fit to the solar splitting, the inverse-form neutrino sector predicts $\Delta m_{21}^2 = 2.37 \times 10^{-3} \text{ eV}^2$ against the measured 2.513×10^{-3} — a -5% deficit, $\approx 2.7\sigma$ in the scale-free ratio after JUNO's first solar-splitting measurement, that JUNO's forthcoming Δm_{21}^2 will decide — with $\Sigma m_\nu \approx 58 \text{ meV}$ against the tightening Planck+BAO bound and $m_1 \approx 0.26 \text{ meV}$. The lepton-nucleon coincidence $\Sigma m_\ell = 1883.0 \text{ MeV}$ against $2m_N = 1877.8 \text{ MeV}$ ($+0.28\%$) is closed or killed by an isospin-symmetric lattice determination of m_N ; the nucleon's positive electromagnetic self-energy moves the QCD-only comparison away from closure, so the pole-mass identity leans toward kill, the

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scheme-corrected reading (self-scale MS-bar leptons) surviving at the 0.1% level with the self-scale caveat. These are the numbers on which the programme stands or falls.

The honest summary is not that mass “comes from geometry” but something sharper: the geometry of circulation forces the *form* of the mass spectrum — its circulant, its ratios, the constancy of Q , the balanced amplitude — while the *scale* is set by the dynamics of a medium (vortex circulation, confinement, one coupling); the *phase* is fixed by the gauge quantum numbers the geometry carries. That same phase, differing across sectors because their quantum numbers differ, does not by itself produce CP violation (circulant sectors give $J=0$; CP violation requires symmetry breaking, outside this construction), while the mixing magnitudes remain a mass-ratio effect outside the geometry — an accurate map of what the circle governs and what it does not. Mass is the circulation energy of a rotor in the source/sink medium, patterned by geometry. Much of the underlying phenomenology — the circulant, the sector pattern, the seesaw extension, the inverse relation — is due to the cited authors; ours are the rotor definition of mass, the hydrodynamic reading of its scale, the boundary map of what geometry does and does not force, and the programme (Sec. 26) that reduces every remaining question to a specific calculation. The complex rotor is the internal parameter of a real mass relation; $\bar{z} = z^{-1}$ on the circle is a fact about that parameter space, carrying no claim about physical space.

Note added (August 2026). Six results obtained after submission are recorded here for the revision record; each is machine-verified and tagged.

(1) The tier factor of the phase law is mechanized. Three vertex-level routes are closed by proven walls: the gauge-invariant dressing of any singlet vertex vanishes identically; $SU(2)$ pseudoreality makes the delta- and epsilon-contractions one channel; broken-phase Goldstone counting carries partner-Yukawa weights. What survives is the condensate count: the high tier grips the quaternionic order parameter (the weak doublet; phase sphere S^3 , three directions with equal weights forced by custodial symmetry), the low tier grips a single complex order parameter (phase sphere S^1 , one direction), and the factor 3 is the ratio of phase-sphere dimensions. With the colour Casimir already derived, every factor of $1/u$ is a counted channel multiplicity. The channel table and the custodial equality have been checked numerically; the two-condensate architecture it rests on is corroborated by the factor itself, since a single shared condensate would count three channels for both tiers and admit no tier factor.

(2) Reading convention, declared. The quark-sector phase and amplitude comparisons in this paper are made with self-scale masses (each mass at its own conventional scale); the lepton sector is convention-free (pole masses; the rigidity theorem). A reader recomputing the quark entries at a single common scale will obtain shifted values; the convention is part of the law’s statement, exactly as the pole convention is part of Koide’s.

(3) The up-sector amplitude anomaly is scale-robust. Its one-loop trajectory runs monotonically away from both $\sqrt{2}$ and $\sqrt{3}$ over the whole range from the top mass to the unification scale, closing the running-displacement escape; the anomaly stands as a genuine

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dial, uniquely correlated with the sector's double channel engagement (colour and the S3 grip).

(4) Unification tests at the master scale. The one-loop Standard-Model couplings do not meet at $M = 2.008 \times 10^{16}$ GeV (spread 8.8 in $1/\alpha$, about 20 percent); the framework's unification is of kind, not of strength. The scale itself, however, survives the decisive experimental test: the order-of-magnitude proton lifetime at M is 10^{37} years, three orders above the Super-Kamiokande bound that excluded minimal SU(5), and Hyper-Kamiokande should observe nothing — the first dated falsifiability statement for the master clock.

(5) Completeness obstruction. Because the exotic states of the 27 complete a grand-unified multiplet, their colour-triplet and doublet components shift the relative running in exactly antiparallel directions; the entire matter content, split at any thresholds in any generation counts with any number of Higgs doublets, moves the triple-meeting point nowhere — it is rigid at 5.05×10^{13} GeV, which is 2.6 dex below the master clock and proton-decay excluded by eight orders. Coupling unification therefore cannot be achieved by mass-splitting within this content; the remaining road is an intermediate gauge stage, a construction the framework does not yet possess.

(6) The E_6 branch selection is two-jawed. Within E_8 , the branches $E_7 \times \text{SU}(2)$, $\text{SO}(16)$ and $G_2 \times F_4$ are excluded by observed parity violation — pseudoreal or real matter admits only vector-like weak interactions (Slansky, Phys. Rep. 79, 1 (1981)) — while the octonionic ladder walks constructively to E_6 .

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Clinical trial registration: This study is not a clinical trial; trial registration: NOT APPLICABLE.

Ethics and Consent to Participate declarations: not applicable.

Consent to Publish declaration: not applicable.

Competing interests: The author declares no competing interests.

Data availability: All numerical inputs are published Particle Data Group values cited in the text; all computed quantities are reproducible from the equations and procedures fully specified in the manuscript. No datasets were generated.

Author contribution: Randeep Chaudhary is the sole author and performed all work.

Appendix A. The reconciled equation set: summary and parameter accounting

This section consolidates the paper's quantitative content into one equation set. Every number below regenerates from the stated equations in a single run; the sector-by-sector empirical status follows the Scope note of Sec. 7.

I. Structure (forced — no data). $J^2 = -I \Rightarrow \pi_1(S^1) = \mathbb{Z} \Rightarrow Q = N/3$ with $N \in \{0,1,2,3\}$ and multiplicities $C(3,N) = 1, 3, 3, 1$. The four sectors are the $N_c + 1$ rungs of the winding ladder on three transverse planes. Tier rule: the parity of N sorts the sectors between the two condensates — even grades (v, u) couple to the electroweak (quaternionic) tier at scale v , odd grades (e, d) to the QCD (complex) tier — consistent with all four observed clock scales (4/4). Combined with the direct/inverse involution this assigns each sector a unique two-bit address: (v , direct) = up, (v , inverse) = v , (QCD, direct) = e , (QCD, inverse) = d . We record the tier rule as a consistent classification, not a derived mechanism.

II. Spectrum (one form, four instantiations). $[\sqrt{m_k}]^{\pm 1} = \kappa[1 + (b/a)\cos(\theta + 2\pi k/3)]$, with $Q_s = 1/(3\cos^2\varphi_s)$ and $\theta = u(1-u)$. The pinning condition is dynamical: $b/a = \sqrt{2}$ and $\theta = u(1-u)$ hold where $y^2_{\text{heavy}} \ll \lambda_H$ (see III). Sector status, with every residual regenerated: charged leptons (direct; $\theta = 2/9$, $b/a = \sqrt{2}$, $\kappa^2 = \Sigma m_\ell/6$) reproduce e to -0.003% , μ to -0.002% , τ to $+0.0001\%$ — zero free parameters beyond the measured mass sum. Neutrinos (inverse; $\theta = 8/81$, $b/a = \sqrt{2}$; single scale fit to the solar splitting) give $m = (0.26, 8.6, 48.6)$ meV as predictions. Down (inverse, common 2 GeV) pins the amplitude only: $b/a = \sqrt{2} \times (1.001-1.002)$, phase free at ≈ 0.190 (3/16 fails; Scope note). Up (direct) pins nothing: three parameters describe three masses — coordinates, not law.

III. The domain boundary. The one-loop non-universal phase drift is $d\theta/d\ln\mu \sim (3/2) \cdot y_h^2/16\pi^2$ and the back-reaction ratio is y_h^2/λ_H . Numerically (y_{heavy} ; drift per decade; y^2/λ_H): neutrino $\sim 10^{-12}$, 10^{-26} , 10^{-23} ; lepton 0.0102, 2.3×10^{-6} , 8×10^{-4} ; down 0.0240, 1.3×10^{-5} , 4×10^{-3} ; up 0.93, 1.9×10^{-2} , 6.7. What the boundary law delivers is the threshold and the ordering: the pattern holds in every sector with $y^2 \ll \lambda_H$ and fails in the single sector where y^2 exceeds λ_H (up, $y^2/\lambda_H \approx 6.7$). It does not compute the magnitudes of the deviations — the deviation-to-coupling ratio varies by more than an order of magnitude across sectors, and the down-sector deviation is dominated by QCD scheme dependence rather than by back-reaction. The up sector is thus not an unexplained exception but the law's own prediction of where pattern yields to dynamics.

IV. Falsifiers. (i) Δm_{31}^2 predicted $2.37 \times 10^{-3} \text{ eV}^2$ against the measured 2.513×10^{-3} — a -5% deficit, $\approx 2.7\sigma$ scale-free following JUNO's first release ($\Delta m_{21}^2 = (7.50 \pm 0.12) \times 10^{-5} \text{ eV}^2$, solar uncertainty 1.55%, from 2.5%); JUNO's forthcoming Δm_{31}^2 decides, through the scale-free ratio $\Delta m_{31}^2/\Delta m_{21}^2 = 31.88$ predicted against 33.56 ± 0.64 measured (2.6σ); m_1 itself is not a JUNO observable and separates the routes by only ~ 1 meV in Σm_ν , so it does not discriminate. Its normal-ordering preference (inverted-ordering p-value 2–2.6%) supports the ordering assumed throughout. (ii) $\Sigma m_\nu = 57.8$ meV — sitting one meV below the conventional normal-ordering floor of ≈ 59 meV, precisely because Δm_{31}^2 is predicted 5%

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low: the two falsifiers are one prediction seen twice. The cosmological bounds now straddle this value: DESI DR1+CMB give < 72 meV and DESI DR2+CMB (Λ CDM) < 64 meV; the tightest multi-probe combinations reach 52–57 meV, nominally squeezing it, while PR4-likelihood and supernova analyses relax the bound to 100–120 meV, and one 2026 joint analysis reports a 2.7σ preference for a positive mass at $\Sigma m_\nu \approx 0.106$ ($+0.050/-0.069$) eV, bracketing the prediction. This is the programme's fastest-moving falsifier. (iii) $m_1 \approx 0.26$ meV. (iv) $\Sigma m_\ell = 1883.0$ MeV against $2m_N = 1877.8$ MeV ($+0.28\%$); an isospin-symmetric lattice determination of m_N closes or kills the coincidence. The direction is already indicated: the electromagnetic contribution to the isospin-averaged nucleon is positive ($\approx +0.6-1.0$ MeV), so the QCD-only nucleon sits below the physical average and the pole-mass gap widens to $\approx +3.2-3.6$ MeV; the pole-mass identity therefore leans toward kill. A scheme-corrected reading — self-scale $\overline{MS}$ -bar lepton masses, giving $\Sigma m_\ell/2 \approx 939.4$ MeV — survives at the $\approx 0.1\%$ level but inherits the self-scale caveat of the down-sector discussion, and we record the coincidence as leaning negative with the boundary drawn here first.

V. Parameter accounting. Forced (representation data): $Q = N/3$, the multiplicities, the circulant form, Q as an angle, the B_2 termination of the holonomy sum, and the tier/involution labels (as classification). Derived where protected: $\theta_\ell = 2/9$, landing at one part in 10^6 . Predictive: $\theta_\nu = 8/81$, currently in 2σ tension and decidable. Input, with proofs of irreducibility: $b/a = \sqrt{2}$ (Proposition 6: no symmetry compatible with the circulant grading can force it; it is reachable only through a non-generic potential) and the four stiffnesses (dimensionless geometry cannot produce an MeV). Failed and bounded: the coloured-sector phases — down at common scale, up scheme-limited. What is unified is the description: one spectral form, one boundary mechanism, one structural ladder — twelve masses, four sectors, three generations, every residual computed and every label declared.

The domain boundary as a depinning criterion: derivation and cross-sector tests

The critical coupling, twice. Section III above states the boundary; it can also be located. Two independent derivations, using only quantities already fixed, place the critical coupling. Potential channel: pinning fails when the grip equals the medium's self-coupling, $y_c = \sqrt{\lambda_H} = 0.361$, i.e. $m_c \approx 63$ GeV. Protection channel: the pattern survives while the accumulated non-universal drift over the tower stays below the phase itself, $(3/2)y^2 N \ln 10 / 16\pi^2 = \theta$, giving $y_c = 0.52$, i.e. $m_c \approx 90$ GeV. The two channels bracket the boundary at 63–90 GeV — and the fermion spectrum straddles the window with no state inside it: $m_b = 4.2$ GeV sits twenty times below, $m_t = 173$ GeV twice above. Nothing was fitted to produce the straddle, and no future discovery can break it: LEP excludes new charged leptons to ≈ 100 GeV and colliders exclude new coloured fermions to the TeV scale, so the window is closed experimental territory. (The scale-insensitivity of Koide-type deviation parameters, on which the protection channel rests, was demonstrated numerically from 1 GeV to 10^{16} GeV by Li and Ma.)

The response curve: linear, then saturated, with a mandatory scale. Below the boundary the one informative sector responds linearly — the down-sector deviation obeys $|\Delta| = 0.36 \cdot g$, an $O(1/3)$ coefficient natural for weak drive, while the lepton and neutrino

deviations sit below their own noise floors (the two-loop QED level computed above, and NuFIT precision, respectively). Above the boundary the response must saturate: linear extrapolation to the up sector predicts $|\Delta| = 2.4$, while $Q \in [1/3, 1]$ bounds $|\Delta| \leq 1/3$ identically; the measured -0.182 sits at 55% of the hard bound. More sharply, the measured values force the response's local logarithmic slope to fall from ≥ 2.15 (lepton to down) to 0.65 (down to up): no scale-free response and no response with slope bounded by unity can fit all four sectors, so an intrinsic threshold scale is mandatory — a steep crossover or a genuine transition. This resolves the variation of the deviation-to-coupling ratio noted in Section III: it is the signature of saturation past a threshold, not noise.

Second-medium instance: the band criterion. The same criterion form — excitation frequency against the medium's gap — operates in the baryon sector, where the pattern object is the N - Δ rotational band and the gap is m_π . With $1/I = 2(M_\Delta - M_N)/3 = 195.4$ MeV read from the splitting itself, the rotational frequencies are $\omega_N = 97.7$ MeV $< m_\pi = 138$ MeV $< \omega_\Delta = 293.1$ MeV $< \omega_{5/2} = 488.5$ MeV. Observed: the nucleon is stable; the Δ is broad ($\Gamma/\Delta M = 0.40$, an order-one width for an order-one overdrive); no narrow $J = 5/2$ band member exists. The single parameter-free content is the inequality pair — m_π falls between ω_N and ω_Δ — and it holds. The radiation condition $\omega > m_\pi$ for rotating solitons is known physics; the identification of that condition with the flavour-sector domain boundary is what is claimed here.

The lattice dial: the crossing, measured. Lattice QCD dials m_π across hundreds of ensembles, converting the single physical point into a curve. The criterion predicts a resonance-to-stable crossing where $M_\Delta - M_N = m_\pi$, i.e. at $m_\pi^* \approx 270 \pm 20$ MeV. The published record: at $m_\pi \approx 200$ MeV (CLS D200) the Δ is a clear resonance with the pole near the $N\pi\pi$ threshold; at $m_\pi = 255.4(1.6)$ MeV (with $m_N = 1073(5)$ MeV) the decay channel is explicitly open, margin $\approx +52$ MeV; at $m_\pi = 280$ MeV the first direct Lüscher determination of the Δ (Andersen, Bulava, Hörz and Morningstar) found the resonance pole close to the N - π threshold — the marginal case, landing on the criterion's knife edge; and for $m_\pi \gtrsim 300$ MeV the Δ is treated throughout the literature as a stable level. The same group's two-ensemble pair ($280 \rightarrow 200$ MeV, one action, one analysis chain) shows the pole migrating off threshold into the open channel exactly as the margin flips sign, in the authors' words in agreement with experiment. Seven independent determinations, every one on the predicted side, with the transition localized within ± 20 MeV of the prediction: the criterion's threshold is a measured function, not a one-point retrodiction. The honest caveat travels with the exhibit: in this sector the criterion coincides with decay-threshold kinematics, so the contribution claimed is the identification, not new dynamics.

The collider face and the confounding. The criterion also has a collider-visible face, as a timescale sandwich: $\Lambda_{\text{QCD}} \ll \Gamma_t \lesssim E_{\text{bind}}$, numerically $0.25 \ll 1.42 \lesssim 1.7$ GeV. Open-flavour top hadrons are forbidden — the top decays about six times faster than the chiral medium can dress it, the depinned rotor unweavable into the low tier's textures — while $\bar{t}t$ Coulombic binding is marginal ($E_b/\Gamma \approx 1.2$), predicting not a narrow state but a smeared threshold enhancement; the $\bar{t}t$ threshold excess consistent with pseudoscalar toponium first reported by CMS in 2025 and since confirmed by ATLAS and again by CMS in the statistically

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independent single-lepton channel (5.1 ± 0.9 pb, Moriond 2026), is that marginal case — observed, twice over. Finally, the confounding must be recorded: the up sector is simultaneously the unique supercritical sector ($g = 6.7$) and the unique even-parity coloured cell of the winding ladder, so existing data cannot separate the dynamical criterion from the discrete selection rule; both may be layers of one account, and the discrimination falls to the falsifiers of Section IV and to the joint-implementation test of the following section.

Relation to Quinta's Spacetime Grand Unified Theory

After this work was developed, we became aware of the Spacetime Grand Unified Theory of Quinta (arXiv:2507.11564), which arrives at a three-mass parametrization structurally identical to the one used here. Quinta writes the square-root masses as $\sqrt{m_k} = (2E/\sqrt{3})|\cos \chi + \sqrt{2} \sin \chi \cos(\delta + 2\pi k/3)|$, obtained as the eigenvalues of a 3×3 Hermitian family operator W with $m_k = \text{eigenvalue squared}$. This is identical to the circulant form $\sqrt{m_k} = \kappa|1 + r \cos(\theta + 2\pi k/3)|$ used here, under the exact and invertible dictionary $\kappa = (2E/\sqrt{3})\cos \chi$, $r = \sqrt{2} \tan \chi$, $\theta = \delta$; at the Koide point $\chi = \pi/4$ both give $r = \sqrt{2}$ and $\kappa^2 = (1/6)\Sigma m_k$. We verify this identity holds to machine precision. We therefore do not claim the parametrization itself as novel: it is the general three-eigenvalue form of a Hermitian family spectrum, and Quinta's derivation of it from the characteristic equation of a Hermitian operator provides a spectral grounding that our hydrodynamic reading motivates but does not replace. As with any three-parameter description of three masses, the parametrization alone carries no predictive content. Our contribution is complementary and lies elsewhere: In its current version (v3, May 2026) Quinta's construction is not silent on these parameters: $\chi = \pi/4$ is obtained in the $\Delta \rightarrow 0$ limit of a $U(3)_F$ -breaking potential (generalising Koide's 1990 model), a limit he notes requires the portal coupling to be exceedingly small — a non-generic potential, which is precisely the route left open by Proposition 6 below, proving that no symmetry can force the value; and the pair (E, δ) is given a modular parameterisation through the Weierstrass invariant and the modular λ -function. What his construction leaves unexplained is the specific value $\delta = 2/9$, which he reports as an observed near-rational (0.222221(2)) and credits, for its prior appearance in circulant mass models, to Brannen and Goffinet — the same lineage the present work descends from. The exchange between the two frameworks is therefore one number for one mechanism: he supplies a tuned-potential mechanism for the amplitude that we lack, and we propose a gauge-quantum-number mechanism for the phase value he lacks — $\delta = \theta = 2/9$ from $u(1-u)$ with $u = 1/3$, the one sector where that mechanism is on firm footing. The amplitude balance and the scale chain of the present work remain motivated but not derived; the empirical status of the phase law beyond the lepton sector is bounded in the Scope note following Proposition 3. Quinta supplies the spectral form; each framework offers the other the one piece of physics it lacks to fix its free parameters.

Acceptance tolerances for a joint implementation. The exchange now carries quantitative tolerances. Any joint implementation in which Quinta's per-flavour scalar sector supplies the up-type texture must output, not fit, three numbers established here: (i) the hierarchy invariant $\ln(m_u/m_t)/\ln(m_c/m_t) = 2.2993 (+0.026/-0.042)$ — any charge assignment linear in the generation index forces the value 2 exactly, excluded at 7.2σ , while $7/3$ lies within 1.3σ ; (ii) the cross-sector deviation exponent $p =$

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$\ln(\Delta_{\text{up}}/\Delta_{\text{down}})/\ln(g_{\text{up}}/g_{\text{down}}) = 0.646 \pm 0.046$, where $g = y^2_{\text{heavy}}/\lambda_H$ — a flavon whose couplings are loop-induced by the fermions themselves ($v_f \propto y^2$) forces $p = 1$, excluded at 7.7σ ; and (iii) a two-phase response: any single smooth saturating response $\Delta(g)$ fitted through the two quark sectors overpredicts the charged-lepton deviation by a factor of 7–30, so the response must contain an intrinsic threshold scale (the local logarithmic slope falls from ≥ 2.15 between the lepton and down sectors to 0.65 between down and up). These three tolerances, together with the mechanism trade above, constitute the acceptance test for the combined framework.

Appendix D. Session provenance, retractions, and the audit trail (v1.5 → v1.6)

Sections 19a–19d, the additions to Sections 24–26, 29 and 31, and this appendix integrate the working-session findings (Unified_Session_Findings v3) into the manuscript. The session's own record went through three versions; what survives is what is printed here, and what did not is listed so that it cannot silently return.

Retracted en route, with reasons: (i) $n_{\text{up}} = 8$ — a common-scale value of a hypersensitive coordinate (0.34σ at M_Z), not present in the law's own scheme, where $n_{\text{up}} = 7.606$; (ii) $1/u_{\text{up}} = 19$ — likewise common-scale at μ^* ; base rate ≈ 1 made it uninformative from the start; (iii) the inverse-frame factor $f = 0.9856(49)$ and the associated frame-dependence pattern — a 2.9σ artefact of evaluating the phase law at a common scale, dissolved by the own-scale scheme in which the down sector needs no factor (0.07σ) and the neutrino deviation stands alone; (iv) the claim that the phase-law/Clebsch conflict had dissolved — it rested on (iii) and returns at 4.2 % in 19d.4. One inequality survives its target: with exact integers (8, 19) the up sector would require $m_u > m_d$, excluded by the neutron–proton splitting alone; the observation is correct and the configuration it excludes is no longer claimed.

Corrections adopted into the standing text: the V8.10 statement that the up sector's departure is driven by top-Yukawa running is quantitatively wrong by a factor of 37 (the drift is +0.6 % against the –22.5 % departure) — the conclusion that no common scale restores the sector stands, the stated mechanism does not; the earlier doublet-ratio form of the colour split (u_d/u_u against N_c) is superseded by Proposition 3 itself, which it approximated; the scope note that the coloured-sector phases are not established is withdrawn per 19a.3. Independently closed during the session, in addition to the routes already recorded in Section 24: the Georgi–Jarlskog Clebsch of 3 ($8.9\text{--}23.5\sigma$, RG-invariant); an additive per-rotor core energy in the shape masses (any $\Delta \sim \Lambda_{\text{QCD}}$ drives $n \rightarrow 3.6$); a tree-level Dirac ripple as the neutrino mechanism (no source); partial inversion and the short Z_3 circle (wrong sign, breaks n); the (W, Z, H) triplet (democratic, $\delta > 1/4$ admits no u); inverted neutrino ordering ($1/u = 1131$).

Provenance: every number in Sections 19a–19d is produced by the scripts of Section 31's addition, run on the stated inputs; the figures of the E_6 /neutrino suite are unchanged. The session record (Unified_Session_Findings v1–v3) is preserved separately and is not part of this manuscript.

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v1.6 → v1.7 (2 September 2026)

Data: inputs moved to FLAG 2024; m_τ to the 2024/25 average (lepton sector 0.4σ). Superseded: the 0.58σ up-sector closure (loose- m_c artefact; now 2.2σ), the $m_u = 2.75$ MeV falsifier (± 1.05 MeV; replaced by $m_c(m_c) = 1.2573 \pm 0.0055$), the 1.6–3.0 GeV floor bracket (now [2.1, 4.5] with N_f -matched running). Retracted: the suggestion that the neutrino phase residual and the lepton mixing require the same Dirac structure — disproved by the polar-decomposition independence theorem (19g). Added: the charge dictionary and number-operator form (19e); the 2T generation of 3, 4, 12 and the composite 9 (19f); tri-bimaximal mixing from the Z_3/Z_2 structure with the TM_1 completion (19g); the even-isospin common factor and the finite-matching cross-check (19h); the five clocks, the branch seesaw with its five tests, and the m_d prediction (19i); the Z_6 spin-generation relation and the representation disambiguation (19f). All numbers reproduce from scripts A–Q; full derivations and provenance in the companion thesis v9, DOI 10.5281/zenodo.22339167.

v1.7 → v1.8 (5 September 2026)

Added: Section 19j (the bosons as the laws' complement: domain statement, the custodial triplet at $k = 0.030$ and $\delta = 0$, the derivative charge dictionary, the coupling-times-condensate mass pattern on both branches, the complete force ledger with the no-new-carriers commitment and the emergent-gravity stance); script S; status addendum. Context recorded: the 2026 convergence on the lepton mechanism — Shulga arXiv:2605.10245 (the Green-dressed family cycle, whose Berry phase $\gamma(\Delta)$ equals $u(1-u)$ identically by the Fourier series of the circle Green function), Rousselle arXiv:2608.19277 — with the four-sector law, the charge dictionary and the frames remaining this programme's distinct content. Companion paper under review at IJQF as 'The Fermion Mass & Charge Spectrum from Unified Equations' (v10, DOI 10.5281/zenodo.22339167).

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