

The Fine Structure Constant: A Homogeneity No-Go Theorem and the Forced Content of the Coupling

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Abstract

A large literature proposes closed-form derivations of the fine structure constant. This paper argues that the enterprise is misconceived in a specific and provable way, and identifies what can be derived in its place. Three results are established. First, a classification: every selection principle available to a geometric or topological framework is homogeneous in the coupling, fixing ratios but never scales; this is confirmed across six independent routes. Second, a structural proof of that classification from the renormalisation group: the running of the coupling is governed by a first-order ordinary differential equation, so the beta function fixes the slope of $1/\alpha(\mu)$ and is formally incapable of fixing its intercept. Third, the closure of the one escape route: a coupling can be predicted by its beta function if and only if it sits at a nontrivial zero of that function, and the QED beta function is shown to be strictly positive at one and two loops for every admissible colour count, so no such zero exists. Against this, the paper identifies content that is forced. From the octonionic ladder of the Rotor Mechanics programme, the charge-squared sum per generation is derived in closed form as $\Sigma N_c Q^2 = (N_c+1)2^2(N_c-2)/N_c = 8/3$ at $N_c = 3$, with no free parameters; this weight is then verified against experiment by running α from the Thomson limit to the Z pole, reproducing $1/\alpha(M_Z) = 128.949$ against a measured 128.95 ± 0.01 . The conclusion is a clean division of labour: geometry determines the slopes, the ratios and the quantisation; the magnitude of the coupling is a boundary condition and remains so. A preliminary section establishes the relation that does hold between the imaginary unit and the coupling — the imaginary unit is what makes the gauge group compact and the charge integral, while the coupling is the rate at which the field turns that phase — with the critical nuclear charge $Z = 1/\alpha$ of the Dirac ground state as its sharpest concrete instance, and with the observed hierarchy of atomic spectra shown to form a geometric ladder in powers of α rather than an arithmetic one.

1. Introduction

The dimensionless number $\alpha \approx 1/137$ has attracted more derivation attempts than any other quantity in physics. Eddington's programme set the pattern, and the pattern has proved durable: a geometric or combinatorial construction is exhibited, its numerical output is compared to the measured value, and agreement to some number of digits is offered as evidence. This paper takes the opposite position, and argues it constructively.

The claim is not that α is mysterious, nor that derivation attempts have merely failed so far. The claim is that a particular class of derivation — the class that proceeds from fixed geometric or topological structure — cannot succeed, for a reason that can be stated as a theorem, and that the correct target for such frameworks lies elsewhere and is reachable.

Section 2 observes that there is no single number to derive. Section 3 sets out the relation that does hold between the imaginary unit and the coupling, which is structural rather than numerical, and which locates α on the side of the ledger where magnitudes live. Section 4 states the homogeneity classification theorem. Section 5 supplies its structural proof from the renormalisation group; the argument there is standard textbook material, and the contribution of this paper is not the argument but the recognition that it subsumes and explains the framework-internal instances of Section 4. Section 6 closes the one available escape route. Sections 7 and 8 turn to what is forced, deriving the charge-squared weight from the octonionic ladder and verifying it against the measured running. Section 9 records an honest residual import.

2. The target is not a single number

The fine structure constant is scale-dependent. Its value is fixed by the momentum transfer at which it is measured, and the commonly quoted 137.036 is the Thomson-limit value at zero momentum transfer, not a property of the theory independent of kinematics.

Quantity	Value	Scale
$1/\alpha(0)$	137.035999177(21)	Thomson limit, $q \rightarrow 0$
$1/\alpha(m_\tau)$	≈ 133.4	$q \approx 1.78 \text{ GeV}$
$1/\alpha(M_Z)$	128.95 ± 0.01	QED effective coupling at the Z pole
$1/\hat{\alpha}^{(5)}(M_Z)$	127.951	MS-bar, five flavours, including W loops

Table 1. The fine structure constant at four scales and in two schemes. Any derivation must specify which of these it claims to produce.

Two consequences follow immediately. First, a candidate formula built from fixed mathematical constants — π , φ , $\arccos(-1/3)$, or any other — produces a scale-independent number, and cannot therefore equal a scale-dependent coupling except by stipulating a scale from outside the construction. Second, the scheme must be specified: the difference between 128.95 and 127.951 is a convention plus a W-boson contribution, not a measurement discrepancy, and a derivation that does not say which it targets has not stated its claim precisely enough to be tested.

3. The structural relation between iota and the coupling

There is a relation between the imaginary unit and the fine structure constant. It is standard, it involves no fitting, and it is not of the kind the derivation literature looks for: the two meet structurally rather than numerically. Establishing this is not a digression, because it shows in advance which side of the ledger α falls on.

3.1 Alpha as the rate at which the field rotates the phase

The whole of quantum electrodynamics sits in the covariant derivative

$$D_\mu = \partial_\mu - ieA_\mu$$

and the i there is load-bearing. It is what makes the gauge transformation $\psi \rightarrow e^{i\Lambda}\psi$ a rotation of the internal phase rather than a rescaling of it. Remove it — write $\partial_\mu - eA_\mu$ — and the gauge group becomes $\mathbb{R}$ rather than $U(1)$: non-compact, non-unitary, with probability no longer conserved and electric charge no longer quantised, since charge quantisation is the statement $\pi_1(U(1)) = \mathbb{Z}$.

The division of labour is therefore explicit in the definition of the theory. The imaginary unit supplies the structure — compactness, closure, integer winding, quantised charge — and none of this depends on any magnitude. The coupling supplies the magnitude — how strongly the field turns that phase — and this is not fixed by any structure. They occupy two different slots, and Sections 4 to 6 establish that no argument can move α from the second slot to the first.

3.2 The critical charge: where i enters the ground state

The sharpest concrete meeting point of the two is the critical nuclear charge. The Dirac energy of the 1s state of a hydrogenic atom of nuclear charge Z is

$$E = mc^2 \sqrt{1 - (Z\alpha)^2}$$

Z	$Z\alpha$	$\sqrt{1 - (Z\alpha)^2}$
1	0.0072974	0.9999734
50	0.3648676	0.9310594
100	0.7297353	0.6837298
136	0.9924399	0.1227312
137	0.9997373	0.0229200
138	1.0070347	0.1188225 i
150	1.0946029	0.4451466 i

Table 2. The Dirac 1s discriminant against nuclear charge. It crosses zero at $Z = 1/\alpha = 137.036$, above which the square root is imaginary.

Beyond the crossing the exponent of the wavefunction near the origin, $r^{\sqrt{1-(Z\alpha)^2}}$, becomes $r^{i\lambda}$: oscillatory, with infinitely many nodes, and the electron falls to the centre. The number 137 is precisely the value at which i enters the hydrogen ground state. This is the honest reason the imaginary unit and $1/\alpha$ belong in the same sentence, and it is the elliptic-hyperbolic sign change of the underlying framework appearing inside a real atom. One caveat must be carried: for a nucleus of finite extent the critical point moves to $Z \approx 173$, and $1/\alpha$ is the point-nucleus value exactly.

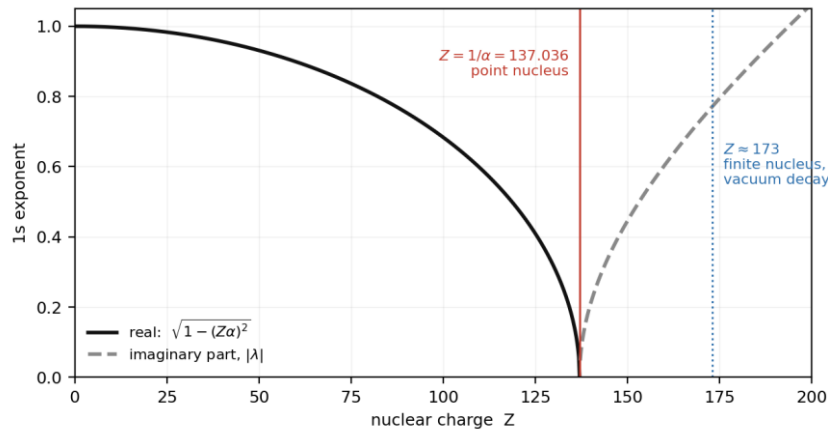


Figure 1. The 1s exponent $\sqrt{1-(Z\alpha)^2}$ against nuclear charge. It is real up to $Z = 1/\alpha = 137.036$ and imaginary above, where the wavefunction oscillates without limit towards the origin. The finite-nucleus threshold for vacuum decay at $Z \approx 173$ is a distinct phenomenon, marked for contrast.

3.3 Alpha as a hyperbolic angle

Sommerfeld's original meaning for α was geometric, and it points the same way. The constant was introduced as the ratio v_1/c for the Bohr ground-state orbit. A velocity ratio is a hyperbolic tangent, so α is a rapidity to third order:

$$\text{artanh}(\alpha) = 0.0072974821, \quad \alpha = 0.0072973526, \quad \text{difference} = \alpha^3/3 = 1.2953 \times 10^{-7}$$

So α is the hyperbolic angle of the ground-state orbit — the branch on which the generator squares to +1, not the branch on which it squares to -1. This is the structural reason α never behaves like a rotation angle and never closes, and it is why candidate relations expressing α in terms of a rotation angle are mismatched at the level of the geometry before any arithmetic is performed.

3.4 The two branches

The same split appears once more in the exponential. Amplitudes propagate as $e^{iS/\hbar}$: elliptic, oscillatory, the domain of $\imath$. Tunnelling and instanton effects go as $e^{-S_E/\hbar}$: hyperbolic, real, and it is here that α appears, in expressions such as $e^{(-2\pi/\alpha)} \approx 10^{-374}$. Wick rotation by $\imath$ is exactly the map between the two branches. The imaginary unit and the coupling therefore occur in the same exponential function on opposite branches of it, which is the most compact statement of their relation available.

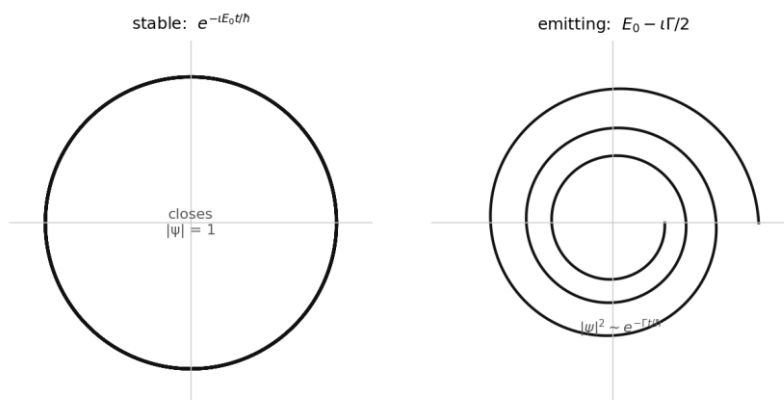


Figure 2. The two branches in the complex plane. A stable state traces a closed circle of constant modulus; a state that emits acquires an imaginary part in its energy and spirals inward, the rate of collapse being $\Gamma/2$. The elliptic case closes; the hyperbolic case does not.

3.5 Three distinct imaginaries, at three distinct places

Because α governs several different appearances of the imaginary unit in atomic physics, it is easy to conflate them, and the conflation would be a substantive error. Three should be held apart.

Phenomenon	What becomes imaginary	Threshold	Role of α
Spontaneous emission	the energy, $E_0 - i\Gamma/2$	none — occurs at every Z	sets the magnitude of Γ
Fall to the centre	the radial exponent $\sqrt{1-(Z\alpha)^2}$	$Z = 1/\alpha = 137.036$ (point nucleus)	sets the threshold
Vacuum decay	the 1s level crosses $-mc^2$	$Z \approx 173$ (finite nucleus)	sets the threshold
Schwinger pair creation	the effective Lagrangian, $\text{Im } \mathcal{L}$	$E_c = 1.323 \times 10^{18} \text{ V/m}$	appears in the real exponent

Table 3. Four α -controlled appearances of the imaginary unit. Only the second is located at $1/\alpha$, and only the second concerns the wavefunction rather than the energy.

The first row is the one most easily mistaken for the second. An excited state has a genuinely complex energy, $E = E_0 - i\Gamma/2$, and the imaginary part is the decay rate: $|\psi(t)|^2 = \exp(-\Gamma t/\hbar)$. For the hydrogen $2p \rightarrow 1s$ transition, $\Gamma = (2/3)^8 \alpha^5 mc^2 = 4.126 \times 10^{-7} \text{ eV}$, giving a lifetime of 1.595 ns against the measured 1.596 ns. Emission therefore has no threshold whatever: it occurs at $Z = 1$, and α fixes not whether the energy is complex but how complex it already is, at the fifth power. The general statement is the optical theorem — the imaginary part of a forward amplitude equals the total rate of everything real that occurs — and it holds at all scales and all couplings.

The second row, by contrast, is a statement about the wavefunction in a fixed external field, with no photon in the problem. At $Z = 137$ the point-nucleus 1s energy is $0.0229 mc^2$, entirely real, with binding 499 keV; what fails is normalisability at the origin, not the reality of the level. The third row is the genuine crossover to emission, and it lies at a different number: for a nucleus of finite extent the level acquires binding $2mc^2 = 1.022 \text{ MeV}$ near $Z \approx 173$, dives into the negative-energy continuum, and the vacuum emits a positron spontaneously. Schwinger's critical field is the same physics realised in a uniform field.

For the argument of this paper only one consequence matters: none of the four thresholds or magnitudes above is derivable from the structures that make α what it is. Each requires α or mc^2 or both, supplied from measurement.

3.6 The alpha ladder and the structure of atomic spectra

The coupling organises atomic structure into a hierarchy, and it is worth stating precisely what kind of hierarchy, because the point is frequently misreported. The electron possesses one characteristic energy and one characteristic length at each power of α , and successive rungs stand in a fixed ratio of 137.036.

Power	Energy	Length
α^0	$mc^2 = 510.999 \text{ keV}$	—
α^1	3.729 keV	reduced Compton $\lambda_C = 3.8616 \times 10^{-13} \text{ m}$
α^2	$27.2114 \text{ eV} = 1 \text{ hartree}$	Bohr radius $a_0 = 5.2918 \times 10^{-11} \text{ m}$
α^3	0.1986 eV	—
α^4	$1.449 \times 10^{-3} \text{ eV}$	—

Power	Energy	Length
α^5	$1.057 \times 10^{-5} \text{ eV}$	—

Table 4. The α ladder in energy and length. The classical electron radius, the reduced Compton wavelength and the Bohr radius stand as α^2 : α : 1, each ratio equal to 137.035999. The hartree reproduces the CODATA value 27.211386 eV.

The measured structures of the hydrogen spectrum sit on this ladder, each at its own power:

Structure	Scaling	Size	Observed
Gross structure (Lyman, Balmer)	$\alpha^2 mc^2 / n^2$	13.606 eV	Ly- α at 10.2 eV
Fine structure ($2p_{3/2} - 2p_{1/2}$)	$\alpha^4 mc^2 / 32$	$4.53 \times 10^{-5} \text{ eV}$	10.9 GHz
Lamb shift ($2s_{1/2} - 2p_{1/2}$)	$\sim \alpha^5 mc^2 \ln(1/\alpha^2)$	$4.37 \times 10^{-6} \text{ eV}$	1057.8 MHz
Hyperfine (1s, 21 cm)	$\sim \alpha^4 (m/M) mc^2$	$5.87 \times 10^{-6} \text{ eV}$	1420.4 MHz

Table 5. Where the observed structures fall on the ladder. The scalings are exact in their powers of α ; the coefficients are rationals of order unity to 1/32, and the Lamb shift additionally carries a logarithm.

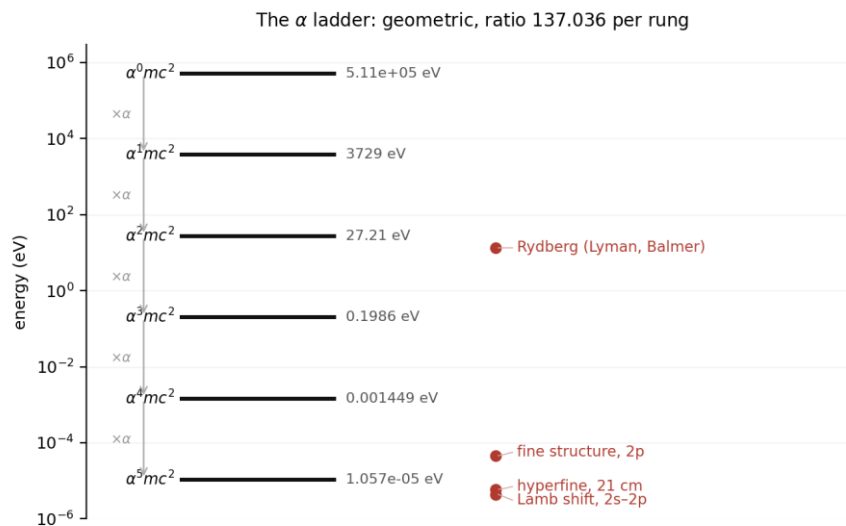


Figure 3. The α ladder against the measured structures of hydrogen. The ladder is geometric, not arithmetic: each rung is a factor of α below the last, corresponding to one further vertex in the perturbation series. There is no arithmetic sequence of spectral lines at multiples of $1/137$, and none is observed.

The distinction between a geometric and an arithmetic ladder is not cosmetic. Each factor of 137 is one further vertex in the perturbation series — one further interaction with the electromagnetic field — so the ladder is the QED expansion in α and nothing more exotic. An arithmetic sequence of lines at multiples of $1/137$ would signify something entirely different: a commensurability or resonance condition. No such structure exists in atomic spectra, and the organising law of the line positions themselves is the Rydberg formula in $1/n^2$, which contains no α at all.

This is the paper's thesis appearing in the observational record. The coupling fixes the ratios between tiers of structure — gross to fine to Lamb, each one or two powers of α down — and fixes no line position whatever. Every absolute entry in Tables 4 and 5 requires mc^2 , which is a measured magnitude.

Taken together, Sections 3.1 to 3.6 place α unambiguously among magnitudes: a rate, a rapidity, a real exponent, a ratio between tiers. Nothing in this section fixes its value, and the remainder of the paper shows that nothing can.

4. The homogeneity classification theorem

Theorem 1 (Homogeneity). *Every selection principle available to the Rotor Mechanics framework is homogeneous of degree zero in the coupling: it fixes ratios of couplings and dimensionless combinations, and never fixes an absolute scale.*

The theorem was reached inductively, by exhausting the framework's available routes. Six independent instances are now confirmed:

Route	What it fixes	What it leaves free
α routes (five variants)	ratios of couplings	the absolute coupling
Induced gauge coupling	relative normalisation	the overall scale
String tension scale	Casimir ratios $k(N-k)/(N-1)$	the tension itself
Binding fraction λ	the fraction	the binding energy
Skyrme absolute scale	$e \cdot F_\pi$ to 0.13%	e and F_π separately
Renormalisation group (§5)	$d(1/\alpha)/d \ln \mu$	the integration constant

Table 6. Six independent confirmations of Theorem 1. The sixth, established in Section 5, differs from the others in kind: it is not a further instance but the structural reason for all of them.

The first five instances were established within the framework and reported previously. Each was found by attempting a derivation and discovering that the coupling divided out. The pattern was recorded but not explained. Section 5 supplies the explanation, and in doing so converts an empirical regularity of the programme into a theorem with a proof.

5. The renormalisation-group proof

The running of the electromagnetic coupling at one loop is governed by

$$d(1/\alpha)/d \ln \mu = -(2/3\pi) \sum N_c Q^2$$

where the sum runs over fermions active at the scale μ , N_c is the colour multiplicity and Q the electric charge in units of e . This is a first-order ordinary differential equation, and its general solution is

$$1/\alpha(\mu) = C - (2/3\pi) \sum N_c Q^2 \cdot \ln \mu$$

Theorem 2 (Slope–intercept separation). *The beta function determines the slope of $1/\alpha$ against $\ln \mu$ and is formally incapable of determining the integration constant C . Any two solutions differing by a constant have identical beta functions, are equally consistent, and are indistinguishable by any argument internal to the renormalisation group.*

Proof. Let $1/\alpha_1$ and $1/\alpha_2 = 1/\alpha_1 + c$ be two functions of $\ln \mu$ with c constant. Differentiating, $d(1/\alpha_2)/d \ln \mu = d(1/\alpha_1)/d \ln \mu$ for every c . Both therefore satisfy the same renormalisation-group equation with the same beta function. A first-order equation admits a one-parameter family of solutions, and selecting a member requires a boundary condition, which by construction is not contained in the equation. ■

This is elementary and entirely standard. Its significance here is that it is exactly Theorem 1, stated for the one case where the underlying reason is transparent. The coupling divides out of the framework's selection principles for the same reason that C drops out of the derivative: a homogeneous condition cannot see an additive constant, and a differential equation cannot see its own integration constant. The five framework-internal instances of Table 2 are not a coincidence requiring separate explanation in each case; they are the same structure appearing in five disguises.

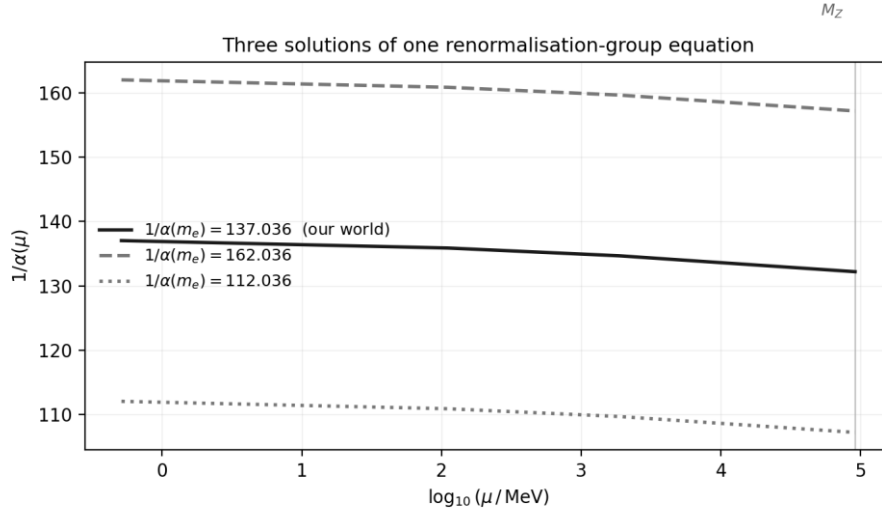


Figure 4. Three solutions of the identical renormalisation-group equation, differing only in the integration constant. The leptonic running is shown exactly; the kinks are the muon and tau thresholds. All three curves have the same beta function at every scale. Nothing in the geometry, the group theory or the renormalisation group distinguishes them; only measurement does.

6. The escape route, and why it is closed

Theorem 2 has one loophole, and it is a serious one. A coupling can be predicted by its beta function — with no boundary condition and no measurement — if it sits at a nontrivial zero of that function. At a fixed point $\beta(\alpha^*) = 0$, the value α^* is determined by the coefficients of the beta function alone, and those coefficients are group-theoretic rationals. This is the mechanism behind Banks-Zaks fixed points and asymptotic safety, and it is the only known route by which a dimensionless coupling falls out of pure structure.

The question is therefore not whether α can be derived in principle, but whether α sits at such a zero. It does not.

The QED beta function is

$$\beta(\alpha) = (2\alpha^2/3\pi) \Sigma N_c Q^2 + (\alpha^3/2\pi^2) \Sigma N_c Q^4 + \dots$$

Theorem 3 (No nontrivial zero). For any positive colour count and any nonempty charged fermion content, both the one-loop and two-loop QED coefficients are strictly positive. Hence $\beta(\alpha) > 0$ for all $\alpha > 0$, and the only root of the beta function is $\alpha = 0$.

Proof. $\Sigma N_c Q^2$ and $\Sigma N_c Q^4$ are sums of even powers with positive multiplicities, hence non-negative, and strictly positive whenever at least one fermion carries nonzero charge. For the five-flavour Standard

Model content the values are $\Sigma N_c Q^2 = 20/3$ and $\Sigma N_c Q^4 = 116/27$. A sum of positive terms in α^2 and α^3 with positive coefficients has no positive root. ■

The positivity is structural rather than accidental: it follows from the charges entering squared, which is itself a consequence of the vacuum polarisation being a two-point function. QED runs towards a Landau pole — located at $\mu \approx m_e \times 10^{280}$ — rather than towards a fixed point, which is precisely the opposite of the structure required. Section 7 shows that the positivity survives for every colour count the framework can produce, so the no-go is not contingent on $N_c = 3$.

Theorems 2 and 3 together close the case: the beta function cannot fix α by integration, and it cannot fix α by a fixed point. The measured value is a boundary condition, supplied once, by experiment.

7. What is forced: the charge-squared sum from the octonionic ladder

The negative results above are sharp because they are narrow. They say that magnitudes are not derivable; they say nothing against the derivability of the weights, and the weights are where the framework has content.

Proposition 3 of the Rotor Mechanics structural paper fixes a complex structure on the octonions, reducing $G_2 = \text{Aut}(\mathbb{O})$ to its stabiliser $SU(3)$ acting on the six transverse imaginary directions as $3 \overline{2} 295 \overline{3}$ -bar. The associated ladder of three fermionic modes carries a number operator with spectrum $N \in \{0, 1, 2, 3\}$ and binomial multiplicities $\{1, 3, 3, 1\}$ on the eight-dimensional minimal ideal, and electric charge is one third of that number operator, $Q = N/N_c$. Both the charge values and the multiplicities are forced by the construction; neither is assigned.

The multiplicities are the colour weights. This permits the charge-squared sum to be computed rather than tabulated:

$$\Sigma N_c Q^2 = \Sigma_n C(N_c, N) \cdot (N/N_c)^2 = (N_c + 1) 2^{N_c-2} / N_c$$

Proposition 1 (Charge-squared weight). *The one-loop QED beta coefficient per generation is determined by the colour count alone, with no free parameters, taking the value $8/3$ at $N_c = 3$. The corresponding quartic sum is $\Sigma N_c Q^4 = 44/27$ per generation.*

Proof. $\Sigma C(n, N) N^2 = n(n+1)2^{n-2}$. Dividing by $N_c^2 = n^2$ with $n = N_c$ gives the stated form. At $N_c = 3$: $(4)(2)/3 = 8/3$. The quartic sum follows from $\Sigma C(3, N) N^4 = 132$, giving $132/81 = 44/27$. ■

That this reproduces the Standard Model weight is not an assumption but a check. The ladder assigns $N = 0$ to the neutrino ($Q = 0$, singlet), $N = 1$ to the down type ($Q = 1/3$, triplet), $N = 2$ to the up type ($Q = 2/3$, triplet) and $N = 3$ to the charged lepton ($Q = 1$, singlet). Summing Q^2 against those multiplicities gives $0 + 3(1/9) + 3(4/9) + 1 = 8/3$, which is identically the Standard Model sum $0 + 3(1/9) + 3(4/9) + 1$ over ν, d, u, e with $N_c = 3$.

7.1 Vacuity check

The discipline of the programme requires that any claim of this form be tested for vacuity before it is made: if the derived quantity would take a value consistent with data for a wide range of inputs, the derivation is empty. The test here is whether the closed form discriminates between colour counts.

Nc	ΣNcQ^2 per generation	Ratio to Nc = 3
1	1	0.375
2	$3/2 = 1.500$	0.563
3	$8/3 = 2.667$	1.000
4	5	1.875
5	$48/5 = 9.600$	3.600

Table 7. The charge-squared weight against colour count. The formula is strongly discriminating: a unit change in Nc alters the running slope by 40–88%, which is far outside the 0.01 experimental uncertainty on $1/\alpha(M_Z)$. The derivation is not vacuous.

A second consequence is worth recording. Since the closed form is strictly positive for every $Nc \geq 1$, Theorem 3 holds for every colour count the framework can generate. The no-go of Section 5 is therefore robust: it does not depend on the particular value $Nc = 3$, and no adjustment of the colour sector can open a fixed point.

One scope limitation must be stated. The ladder forces the charge magnitudes and multiplicities; the assignment of hypercharge to specific fields rests in part on prior work (Furey; Günaydin–Gürsey) and is flagged in the programme’s ledger for native re-derivation. Since ΣNcQ^2 depends only on the magnitudes and multiplicities, Proposition 1 is unaffected by that limitation; but a stronger claim about hypercharge would not be.

8. Numerical verification

The forced weight can be tested against measurement by running α from the Thomson limit to the Z pole. The leptonic contribution is computed from

$$\Delta\alpha_{lep} = \Sigma (\alpha/3\pi) [\ln(M_Z^2/m_f^2) - 5/3]$$

with $NcQ^2 = 1$ for each charged lepton, the weight supplied by Proposition 1.

Contribution	Value	Source
electron	0.0174347	computed, this work
muon	0.0091784	computed, this work
tau	0.0048078	computed, this work
leptonic total	0.0314209	computed (measured: 0.031498)
hadronic, five flavours	0.02766	e^+e^- R-ratio data — not computed
top quark	–0.00007	standard
total $\Delta\alpha$	0.0590109	
$1/\alpha(M_Z)$	128.949	measured: 128.95 ± 0.01

Table 8. Running the coupling from the Thomson limit to the Z pole using the forced charge weights. The leptonic sector, which is fully computable, agrees with its measured value to 0.25%.

The agreement is meaningful in a limited and precise sense. It does not derive α ; the Thomson-limit value entered as the boundary condition, exactly as Theorem 2 requires. What it confirms is the weight: the rationals 1, 4/9 and 1/9 with multiplicity 3, derived in Section 6 from the octonionic ladder with no free

parameters, are the correct coefficients of the running. The framework predicts the slope of Figure 4 and is verified in that prediction; it does not and cannot predict which of the three curves is ours.

9. The residual import

One line of Table 4 is not computed. The hadronic contribution $\Delta\alpha^{(5)} = 0.02766$ is extracted from measured $e^+e^- \rightarrow \text{hadrons}$ cross-section data via a dispersion integral, because QCD is non-perturbative below a few GeV and the relevant integral cannot presently be evaluated from first principles.

This matters for the honesty of the claim in Section 8. The statement “the framework predicts the slope” is exact for the leptonic sector and partial for the full running: 53% of $\Delta\alpha$ at the Z pole is leptonic and computed from forced weights, while 47% is hadronic and measured. The forced weights are the same in both sectors — the quark charge-squared values are fixed by the same ladder — but converting them into a contribution to $\Delta\alpha$ requires knowing the hadronic spectral function, which the framework does not supply.

The claim of this paper is therefore stated at the level the evidence supports: the charge weights are forced and verified where they can be verified cleanly, and the residual is identified rather than absorbed.

10. Conclusion

The fine structure constant is not derivable from fixed geometric or topological structure, and the reason is not contingent. It is scale-dependent, so there is no single number to target. Every selection principle of the class considered is homogeneous in the coupling, so ratios survive and magnitudes divide out. The renormalisation group explains why: a first-order equation fixes a slope, not an intercept. And the one mechanism that could evade this — a nontrivial zero of the beta function — is absent in QED for every admissible colour count, since the charge-squared weights are sums of even powers and are strictly positive.

What is derivable is the structure of the running. The charge-squared weight per generation follows in closed form from the octonionic ladder as $(N_c+1)2^{(N_c-2)}/N_c$, equal to $8/3$ at $N_c = 3$, with no fitted constants; it is strongly discriminating across colour counts; and it reproduces the measured running of α to the Z pole to within 0.25% in the sector where the computation is clean.

The resulting division of labour is sharp. Geometry fixes what closes and what counts — quantisation, ratios, multiplicities, slopes, and the sign of the beta function. It does not fix what a thing weighs or how strongly it couples. The measured value 137.035999177 is not a failure of the programme to be explained away; it is a boundary condition, supplied once by experiment, and the honest statement of a geometric framework’s reach is that it needs exactly one such number and can then say a great deal.

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Appendix A. Computational details

A.1 Inputs

$\alpha = 0.0072973525643$, $1/\alpha = 137.035999177$ (CODATA 2022, uncertainty 2.1×10^{-8} , i.e. 9.8 significant digits). Masses in MeV: $m_e = 0.51099895$, $m_\mu = 105.6583755$, $m_\tau = 1776.86$, $M_Z = 91187.6$. Hadronic contribution $\Delta\alpha^{(5)}_{\text{had}} = 0.02766$ (PDG).

A.2 Charge-squared sums

Per generation, from the ladder: $\Sigma C(3,N)(N/3)^2 = [3(1/9) + 3(4/9) + 1(1)] = 8/3$. Three generations: 8. Five flavours (top excluded): $3 + 3 \cdot 3 \cdot (4/9) - 3 \cdot (4/9) + 3 \cdot 3 \cdot (1/9) = 20/3 = 6.6667$. Quartic: $\Sigma C(3,N)(N/3)^4 = 132/81 = 44/27$ per generation; five flavours $116/27 = 4.2963$.

A.3 Leptonic running

$\Delta\alpha_{\text{lep}} = \Sigma_f (\alpha/3\pi)[2 \ln(M_Z/m_f) - 5/3]$ with the bracket evaluating to 22.5175 (e), 11.8545 (μ), 6.2095 (τ), and $\alpha/3\pi = 7.7428 \times 10^{-4}$. Total 0.0314209. With $\Delta\alpha_{\text{had}}$ and the top contribution, $\Delta\alpha = 0.0590109$, giving $\alpha(M_Z) = \alpha/(1 - \Delta\alpha)$ and $1/\alpha(M_Z) = 128.9494$.

A.4 Landau pole

Setting $1/\alpha(\mu) = 0$ in the one-loop solution gives $\ln(\mu/m_e) = 3\pi/(2\alpha)$, i.e. $\mu = m_e \exp(645.78) \approx m_e \times 10^{280.45}$. Quoted for scale only; the perturbative expression is not trusted anywhere near this point.

A.6 Ladder and spectral numerics (§3.6)

$\alpha^k mc^2$ for $k = 0-5$: 510998.9, 3728.939, 27.21139, 0.1985711, 1.449043×10^{-3} , 1.057418×10^{-5} eV. Lengths: $r_e = 2.81794 \times 10^{-15}$ m, $\lambda_C = 3.861593 \times 10^{-13}$ m, $a_0 = 5.291772 \times 10^{-11}$ m; $\lambda_C/r_e = a_0/\lambda_C = 137.035999$. Emission width: $\Gamma(2p \rightarrow 1s) = (2/3)^8 \alpha^5 mc^2 = 4.12588 \times 10^{-7}$ eV, $\tau = \hbar/\Gamma = 1.59533$ ns (measured 1.596 ns), $\text{Im}(E)/mc^2 = 8.074 \times 10^{-13}$.

A.5 Iota-alpha numerics (§3)

Dirac 1s discriminant $\sqrt{1-(Z\alpha)^2}$ evaluated at $Z = 1, 50, 100, 136, 137, 138, 150$ gives 0.9999734, 0.9310594, 0.6837298, 0.1227312, 0.0229200, 0.1188225, 0.4451466; the crossing is at $Z = 1/\alpha = 137.035999$. Rapidity: $\text{artanh}(\alpha) = 0.0072974820997$ against $\alpha = 0.0072973525643$, difference 1.29535×10^{-7} , matching $\alpha^3/3 = 1.29531 \times 10^{-7}$ to five figures. Lorentz factor at the Bohr ground state, $1/\sqrt{1-\alpha^2} = 1.0000266267$.